

# The Creative Genius of Isaac Newton

(The Simple Case)

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There are several geniuses in the field of Renaissance astronomy and physics known to almost every educated person – Copernicus, Galileo, Kepler, and Newton among them. What was the creative genius of these individuals? While it would take an encyclopedic book to be able to answer that question, the current paper is much more modest. It is the purpose of this paper to explain in very few words the creative genius of Isaac Newton in relation to his formulation of the theory of gravitation. (It is called here a theory rather than a law because of the empirical evidence that has been built up over the years in verification of the hypothetical form of the mathematical formulation that was not derived from experiment – e.g., a law.)

The story of Newton sitting under an apple tree one day seeing an apple fall and thinking about the form of gravitation is probably apocryphal. Nonetheless, it could have occurred to Newton that the fall of an apple is not unlike that of the fall of the Moon as it orbits the Earth. It was the fact that he was able to understanding the relationship between the Moon and the apple that constitutes the real creative genius of Isaac Newton. Couched in modern SI terms, this is what Newton did....

He realized that the acceleration of, say, an apple near the surface of the Earth was  $9.8 \text{ m/s}^2$  (in modern terms). That is,

$$a_{\oplus} = 9.8 \frac{m}{s^2}$$

He then calculated the centripetal acceleration of the Moon in its orbit around the Earth by using an equation first provided by the Dutch scientists of his day:

$$a_{\lrcorner} = \frac{v^2}{r}$$

The speed of the Moon's motion was easily derived from the relationship into which he put the proper values for the orbital radius of the Moon and its sidereal period (both known with a high degree of precision in his day)

$$v = \frac{d}{t} = \frac{\text{circumference}}{\text{period}} = \frac{2\pi r}{P} = \frac{2\pi(384,000,000m)}{2,360,000s} = 1020m/s$$

Using the equation for centripetal acceleration, he then came up with the value of the Moon's acceleration

$$a_{\lrcorner} = \frac{(1020m/s)^2}{384,000,000m} = 0.00271m/s^2$$

He then compared the acceleration of objects near the Earth's surface with that of the Moon in orbit and found

$$\frac{a_{\oplus}}{a_{\lrcorner}} = \frac{9.8m/s^2}{0.00271m/s^2} = 3600 = 60^2$$

In this case, 60 represented the radius of the Moon's orbit in Earth radii. From this comparison, Newton was able to conclude that the acceleration of the Moon in its orbit was inversely proportional to its distance from the center of the Earth squared. That is,

$$a_{\lrcorner} \propto \frac{1}{r_{\lrcorner}^2}$$

Given the fact that  $F=ma$ , Newton concluded that the force required to hold the Moon in its orbit around the Earth (and any planet in orbit around the Sun) was similarly dependent upon distance squared. That is,

$$F \propto \frac{1}{r^2}$$

Because gravity is responsible for the perceived weight of objects, and is proportional to the mass of the object,  $m$ , (and supposedly the mass of the gravitating body,  $M$ ), Newton was able to hypothesize that,

$$F \propto \frac{Mm}{r^2}$$

Inserting the proportionality constant,  $G$ , gives us the familiar form of Newton's formulation.

$$F = \frac{GMm}{r^2}$$

So, it should be evident from this deduction that Newton's act of creative genius was in the fact that he was able to use observational evidence to formulate a hypothetical relationship for the nature of the central gravitational force required to keep objects in orbital motion. Now, how could this hypothesis be tested? A generation earlier, Johannes Kepler formulated three planetary laws of motion based upon observation of the planet Mars. He stated these laws thusly:

1. Planets move in elliptical orbits around the Sun with the Sun located at one of the foci.
2. The radius arm between a planet and the Sun sweeps out equal areas in equal time intervals.
3. The period of a planet expressed in years squared equals the semi-major axis (mean radius) of the orbit expressed in astronomical units cubed. That is,

$$P^2 = r^3$$

If the units are arbitrary (e.g., SI units), then the form of the equation would be

$$P^2 = kr^3$$

where the value and units of  $k$  would depend upon the units employed in the equation's other variables. At this point Newton, with his new formulation of gravity and his own second law, was able to write

$$F = ma = \frac{mv^2}{r} = \frac{GMm}{r^2}$$

Substituting for  $v$  ( $2\pi r/P$ ) and canceling, Newton arrived at the following relationship using the two rightmost components of this equation. That is,

$$P^2 = \frac{4\pi^2 r^3}{GM} = kr^3$$

which is Kepler's third or harmonic law! Newton's formulation of the law of gravity therefore was able to explain why the harmonic law was of the nature derived by Kepler – it's because gravity has an inverse-squared nature. Hypothesis then, with this firm underpinning, was on its way to becoming theory.

Note that this formulation is for the simple case that assumes purely circular motion. In reality, the solar system's moons and planets move with elliptical and barycentric motion. Taking both of these considerations into account, Newton was able to derive a more precise form of the Harmonic law

$$(M + m)P^2 = \frac{4\pi^2 r^3}{G}$$

where  $M$  and  $m$  are the masses of the bodies in, say, SI units. If  $M$  and  $m$  are expressed in solar mass units,  $P$  in years, and  $r$  in astronomical units, then the equation simplifies to

$$(M + m)P^2 = r^3$$

This relationship was used to measure the masses of various solar system bodies in solar mass units centuries before the space age. Fly-bys of moons and planets with interplanetary spacecraft have verified this relationship, and provide additional evidence that Newton's formulation of gravity is correct. Additionally, sending spacecraft on interplanetary missions using Newton's formulation of gravity as a guide has proven to be extremely accurate, showing once again that Newton was correct in his

hypothetical assertion of the nature of gravity. The hypothetical form of the law of gravitation has become well-substantiated theory due to the vast amount of empirical evidence that supports it.

It should be noted, too, that Newton's more detailed analysis of the central force problem resulted in a prediction of elliptical motion – precisely what Kepler observed. That is, when gravitational force is assumed to drop off with an inverse-square of the distance, then elliptical motion results. This formulation is beyond the scope of this book. Kepler's law of equal areas is also readily explained by Newton's formulation of gravity.