Employing active learning to establish an empirical basis for kinetic energy
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Physics teachers often introduce the study of kinetic energy by stating without explanation or justification that kinetic energy is equal to one half the product of the mass and squared velocity. If a basis for this formulation is given at all, it often leaves students confused. Any reasonably skeptical student of physics would want to inquire as to the physical reason why kinetic energy is so defined. The simplest answer is that work and kinetic energy so defined are conserved in certain situations. Is there a laboratory activity that physics teachers might employ to help introductory physics students understand that kinetic energy is indeed proportional to mass and squared velocity? Fortunately, the answer is yes.

What follows is the outline of an introductory physics laboratory that has produced excellent results relating potential energy to an experimentally derived definition of kinetic energy. By dropping metallic balls of varying mass (diameter appears to be irrelevant) from a constant height and counting the number of droplets of water required to fill a pit created in a clay “target,” it is possible to show that the (kinetic) energy required to produce each depression is directly proportional to the work of raising a mass of the falling ball. (See Figure 1.) By dropping a ball of known mass from different heights (again, the diameter appears to be irrelevant), it is possible to show that the kinetic energy contained in each release of the ball is proportional to the speed at impact squared. (See Figure 2.) From this evidence it is possible to conclude that:

\[ KE_{\text{impact}} \propto mv^2 \]

Deriving the constant of proportionality can be done algebraically by referring to the kinematic formula employed to calculate the speed of impact.

\[ v^2 - v_o^2 = 2gh \]

From this is derived the equation for speed at impact assuming \( v_o = 0 \).

\[ v = \sqrt{2gh} \]

Squaring both sides of this latter equation and multiplying by \( m \) results in

\[ mv^2 = 2mgh \]

At this point, it is possible to realize that the work required to raise each ball of mass \( m \) to the release height \( h \) above the clay target in the Earth’s gravitational field \( (PE_{\text{release}} = mgh) \) can be separated from the factor of 2, yielding the now familiar relationship

\[ \frac{1}{2} mv^2 = mgh \]

or by definition

\[ KE_{\text{impact}} = PE_{\text{release}} \]

It is important to note that this process points to the principle of conservation of energy and might serve as the first encounter with the concept for many students. Perhaps just as important is the conclusion that certain kinematics laws take the form they do because of conservation of energy.

This experiment has been conducted successfully with college students, and 5th through 8th grade school children and their parents. They have used metal spheres with diameters ranging from 2cm to 5cm, and with masses varying from 30g to 470g. Release heights for these balls range from about 50cm to as much as 2m. Larger masses and release heights are to be preferred in order to minimize the relative error in volume measurement. Once the deformation appears in the clay target, younger students can use water and an eyedropper to measure the volume of the pit. They count the number of droplets of water required to fill the depression completely. Soap is added to the water to reduce surface tension and alleviate any problems with a meniscus. Multiple independent measurements of the volume are important for minimizing error using the water drop method.
An alternative approach for determining the volume of the depression is to use direct measurements along with the following mathematical formula:

\[ V = \frac{\pi x}{6} \left( 3r^2 + x^2 \right) \]

where the depth of the pit at its center is \( x \), and the radius of the pit is \( r \). The value of \( r \) can be determined readily from the semidiameter of the pit. The value of \( x \) is hard to measure directly with ordinary lab instruments, but its value can be found indirectly from the following relationship:

\[ x = R - \sqrt{R^2 - r^2} \]

where \( R \) equals the radius of the metal sphere.

The clay target is typically 1 cm to 2 cm in thickness with a horizontal upper surface. Because clay will change its deformation properties with varying temperature and moisture content, it is important that the conditions of the clay be similar each time a ball drop is performed. Students are asked to minimize the manipulation of the clay by hand, and are directed to use a block of wood to flatten out the clay in preparation for the next ball drop. Data collection is typically conducted over a very short time suggesting that moisture loss due to evaporation isn’t a significant concern. Students are also directed to remove water in a pit with a blotter before reshaping the clay. Observing these precautions, this experiment has been conducted several times with good results (e.g., as evidenced in the exponent values of typical graphs). Use of oil-based plasticine might be another way to minimize drying or wetting of the target.

Conducting this lab activity will provide students with an empirical basis for the definition of kinetic energy. It can be used as an opportunity to conduct a simple, yet meaningful, inquiry-oriented lab activity with introductory-level physics students. The activity can be completed in about an hour. Adding this activity to a study of the relationship between the length of a simple pendulum and its period of oscillation allows the author to teach a paradigm lab titled “The Pit and the Pendulum” that has a certain appeal for those familiar with the work of Edgar Allan Poe.

Graph 1. The effect of ball mass on the volume of the pit. The volume of each pit (which is itself proportional to the kinetic energy required to produce it) is directly proportional to the mass of the falling ball. A physical model requires the regression line to pass through the origin. Data are averages collected by 5th through 8th grade school children working with their parents.

Graph 2. The effect of impact speed on the volume of the pit. The volume of each pit (which is itself proportional to the kinetic energy required to produce it) is directly proportional to the speed of the impacting ball squared. A physical model requires the best-fit curve to pass through the origin. Data are averages collected by 5th through 8th grade school children working with their parents.