Reconciling Experimental with Theoretical Relationships

When students graph relationships between variables in relationships that contain known proportionality constants such as "*g*", how does one extract such terms from the proportionality constant if it is know to be present?

Consider the traditional pendulum lab conducted in a region where the acceleration due to gravity is 9.810m/s^2. When students graph period squared (t^2) versus length (l) to get a linearized relationship, they find a proportionality constant of approximately 4.024s^2/m. That is,

$$t^2 = \left(4.024\frac{s^2}{m}\right)l$$

Writing this in the more traditional form yields the following version of the relationship:

$$t = 2.006\frac{s}{m^{1/2}}\sqrt{l}$$

Now, how does one extract "*g*" from this experimentally determined constant so that students can reconcile the relationship with the theoretically derived form that frequently appears in textbooks as follows?

$$t = 2\pi\sqrt{\frac{l}{g}}$$

That is, how can one logically show that $2.006s/m^{1/2} = 2\pi/\sqrt{g}$ in this case? The answer resides with the use of dimensional analysis. Consider the following dimensional analysis assuming t is a function of l and g and starting with the assumed form of the relationship:

$$t \propto l^x g^y$$

$$s^1 \propto m^x\left(\frac{m}{s^2}\right)^y$$

$$m^0 s^1 \propto m^{x+y}s^{-2y}$$

Now, equating exponents on s and m respectively one finds:

$$1 = -2y \qquad 0 = x + y$$

$$y = -\frac{1}{2} \qquad \text{and} \qquad 0 = x - \frac{1}{2}$$

$$x = \frac{1}{2}$$

Inserting the exponents of x and y and replacing the proportionality with an equality sign and proportionality constant in the assumed form of the relationship $t \propto l^x g^y$ yields

$$t = const\sqrt{\frac{l}{g}}$$

Now, gathering all constant terms in this equation yields:

$$t = \frac{const}{\sqrt{g}}\sqrt{l}$$

Equating the two sets of constants in the experimental and theoretical forms of the relationships respectively, one gets:

$$2.006\frac{s^2}{m} = \frac{const}{\sqrt{g}}$$
$$const = 2.006\frac{s^2}{m}\sqrt{g}$$
$$const = 2.006\frac{s^2}{m}\sqrt{\frac{9.810m}{s^2}}$$
$$const = 6.283$$
$$const = 2\pi \quad (approximately)$$

Hence, replacing the experimentally determined proportionality constant $2.006s^2/m$ with $2\pi/\sqrt{g}$ yields an experimental formula that is completely consistent with the theoretical version.

$$t = 2\pi\sqrt{\frac{l}{g}}$$