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WHERE IS THE MOON TONIGHT?

WHEN STUDENTS ARE ASKED TO observe the Moon at the same time every night for a month, they remark on one of their first surprises: "The Moon wasn't there last night," even though "it was there the night before." During the course of a six-week investigation, seventy-five fifth-grade students discovered the reason for not always seeing the Moon at the same time in the same place. After students began a study of the rotation of Earth, they studied the phases of the Moon and eclipses and explored the relationships of positions of the Earth, Moon, and Sun through guided inquiry. Throughout the investigation, they worked with two-dimensional and three-dimensional representations, translating back and forth between the two; developed units to measure three-dimensional angles; worked with data analysis while gathering and organizing data for a class Moon chart; and created and used multiple representations while making sense of physical phenomena.

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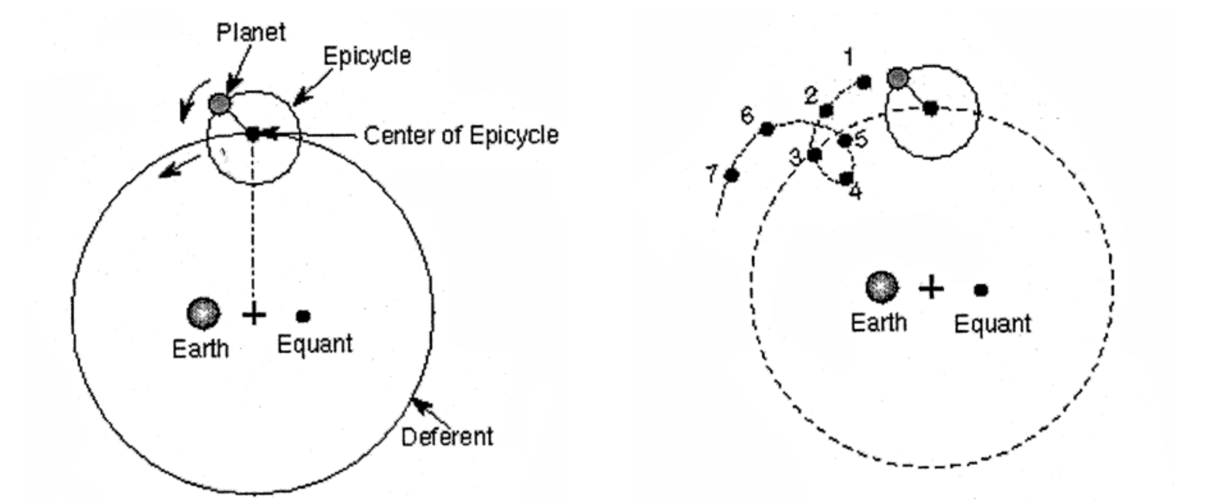
One of the most elementary foundations of science involves going beyond merely observing the world to creating a model of the essentials of what has been observed so as to understand and predict the phenomenon. To confirm the model, we have to see that it matches the data we have, that it makes sense, and that its essentials are simple and elegant. The test of elegance is in the mathematics of the model. For example, when the ancient Greeks observed the movement of the Moon, Sun, and planets, Eudoxus (408–355 BC) proposed a model in which Earth was at the center of a set of concentric transparent spheres on which the planets, Sun, Moon, and stars revolved. This model left many observations unexplained, including the fact that sometimes the planets seemed to move backward. During the second century AD, Ptolemy, the last great ancient Greek astronomer, produced a model that took into account this retrograde motion. He devised the solution of *epicycles*, with planets making small circular orbits while continuing to orbit around Earth. (The astron-



Center of epicycle moves counterclockwise on deferent, and epicycle moves counterclockwise. Epicycle speed is uniform with respect to equant. The combined motion is shown at right.

Deferent motion is in direction of point 1 to 7 but planet's epicycle carries it on cycloid path (points 1 through 7) so that from points 3 through 5 the planet moves backward (retrograde).

Fig. 1 An epicycle is a circular path that itself is circular.

omy Web site, www.astronomynotes.com/history/epicycle.htm, contains excellent animation of epicycles.) To model epicycles mathematically requires more than forty parameters:

> By adding properly sized epicycles to Eudoxus's spheres and adjusting their speeds and rates of rotation, Ptolemy was able to devise a geometrical model that came reasonably close to predicting the actual planetary motions. Where discrepancies still existed, Ptolemy added smaller rotating circles that traced out epicycles upon some of the prior epicycles. (Zabrowski 1999, p. 90)

Furthermore, to these parameters Ptolemy had to add a different way of looking at Earth as not being the true center of each planet's motion, as illustrated in **figure 1** (Strobel 2004) for one planet. Ptolemy obtained these parameters by trial and error, so he had to continue to adjust them to conform to what was observed from Earth. In 1543, Polish astronomer Copernicus published *De Revolutionibus Orbium Coelestium* (On the Revolutions of Celestial Bodies), hypothesizing that the planets move around the Sun in circular orbits, but he still needed fifteen independent parameters to make his model agree with his observations. Because Johannes Kepler believed that God created a perfect universe, he expected all the planets to have circular orbits about the Sun. During the early seventeenth century, he spent years trying to fit this model mathematically to match the data that had been carefully observed by Tycho Brahe in the late sixteenth century (Hellemans and Bunch 1988). After working on it for eighteen years, he had formulated three laws of planetary motion: The planetary orbits are elliptical with the Sun at a focus of the ellipse, the line between the Sun and a planet sweeps through equal areas in equal time intervals, and the ratio of the cube of the semimajor axis to the square of the period of revolution is the same for each planet. Based on these three laws, the mathematical model takes into account all the data. It is elegant both geometrically and algebraically, requiring only eleven parameters for the model, which are necessary to describe the size and shape of each elliptical orbit. In contrast to Ptolemy's trial-and-error method, these three laws were established by direct calculation from physical measurements of position and time (Zabrowski 1999). (For a good explanation of Kepler's Laws, see a site maintained by the University of Tennessee—Knoxville, csep10.phys.utk.edu/astr161/lect/history/kepler.html.)

Because astronomy is a domain in which mathematics has been the framework and language for the scientific models, it is a natural place for integrating mathematics and science. In this article, we discuss the unit in which the fifth-grade space cadets investigated the Moon and its phases. This scientific inquiry provided a rich, natural context for creating two-dimensional and three-dimensional models for understanding. The mathematics provided the language and tools necessary for a deeper analysis of the science concepts and applications. This article describes Kevin Cornell's experiences conducting this activity with his fifth-grade class at a Midwestern elementary school.

Students use overhead-projector light to simulate light on the Moon.

Students use a globe to explore Earth's rotation on its axis.

The Teacher, the Setting, and Preliminary Work

AFTER EXPERIENCING AND REFLECTING ON HOW children learn, I (Kevin Cornell) have become a teacher who uses inquiry to teach science. Through pretesting my students, I found out what conceptions they held about the Moon and Earth and used their understandings and misunderstandings as a starting point for the investigations. Students participated in lessons that I designed so that they could test and investigate their existing understanding through meaningful activities and scientific discourse.

Misconceptions

WHILE PRETESTING, I DISCOVERED THAT STUDENTS began the unit with misconceptions about the Moon and Earth. This finding is supported by researchers who have found that students in advanced college classes, as well, have misunderstandings about the Moon and its phases (Trumper 2001, p. 1115). Common misconceptions, shared by my students at the start of the unit, include the cause of night and day on Earth, an incomplete or confused understanding of the cause of the Moon's phases, the terms *phases* versus *eclipses*, the Moon's rotation and its orbit, what causes the light and dark parts of the Moon, and when someone can see different phases of the Moon (Barnett and Morran 2002; Taylor, Barker, and Jones 2003; Trumper 2001). Furthermore, many of the fifth-grade students believed that the Moon is only out at night, and that the Moon is a flat object. Many who believed that Earth rotates westward, instead of eastward, on its axis also had trouble believing that half of Earth is in darkness (night time) and that the other half is in the light (day time).

The guided-inquiry Moon Unit leads students through explorations of the concept of day and night, the Moon's orbit around the Earth, the phases of the Moon, and solar and lunar eclipses. This article discusses the essential components of the unit and multiple related activities and tasks.

The Moon, Three-Dimensional Geometry, and Measurement

ABOUT ONE MONTH BEFORE STARTING THE Moon Unit in the classroom, I assign the Moon journal to my students. Each Moon journal contains thirty copies of the diagram in **figure 2**, on which students document their observations. Students are responsible for observing the Moon at the same time each evening. Many of them are quite startled when the Moon is not in the same place on sequential nights and even more so when the Moon is not visible at all.

Each observation must include the altitude of the Moon and the shape of the visible part of the Moon. The first few Moon observations are very difficult for students, not only because they are not sure how to note exactly where the Moon is, including its altitude, but also because they may have forgotten to look or they may have left their journals at school. After the students have been working on their journals for a few days, I check them and discuss a few Moon observations with the students. This allows students to check their first few observations. If students are un-

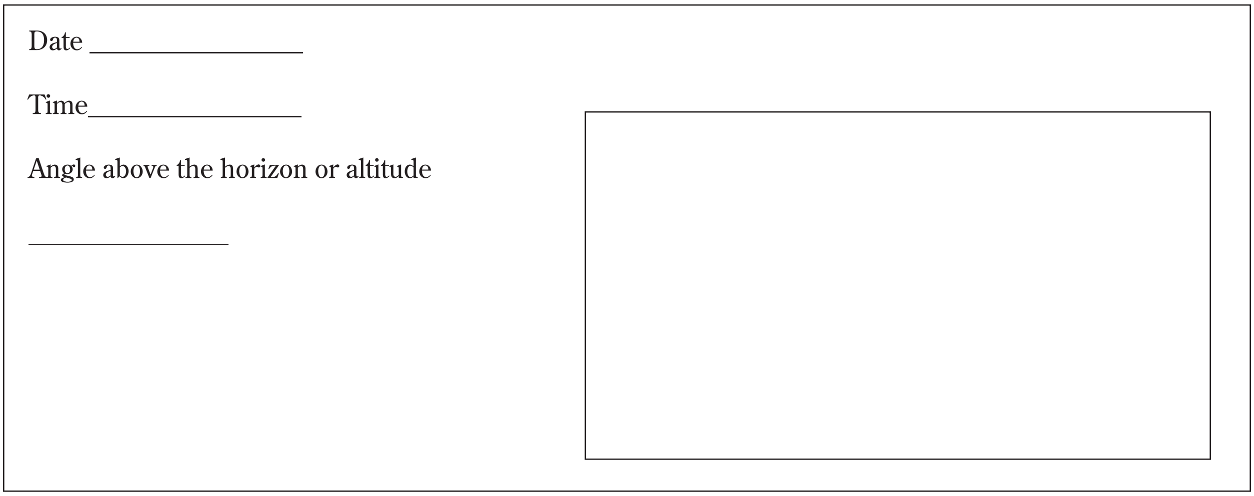


Fig. 2 Students are asked to use this diagram when making entries in their Moon journals. Each entry is to be made on a separate page of the journal. Students are to draw their observations of the Moon in the smaller box.

able to make consistent observations at home, a classwide observation can be made each morning when students arrive and line up to go inside.

I randomly check their journals five or six times throughout the four weeks of observations. I do a Moon observation each day and night and check students' journals on dates that I know had good possibilities for Moon observations. Because I live in the same area as my students, I know what they should have seen. When the sky is cloudy, students simply write "cloudy" in their journal. Later in the unit when they begin comparing their data, students are able to log on to the Web and look up what Moon phase was visible when it was cloudy. This could be an ideal time to encourage students to communicate with others in different parts of the country who perhaps are not experiencing cloudy conditions on the same days.

Another problem is measuring the altitude of the Moon. To do so can be tricky without using appropriate tools, but with help, the students develop a unit for measurement. Students make two fists and stick their arms straight out in front of them. From a horizontal position, they stack their fists until they create a 90 degree angle with their arms stretched completely above their heads. Then they divide the 90 degrees by the number of fists to determine the number of degrees in a fist. Students' fist sizes will vary; however, for their own Moon journal, they can use their fists to measure the altitude of the Moon above the horizon. This method of measurement will naturally produce variations among students, but it presents an opportunity to compare the students' units of measurement and discuss the reason for standard measurement units.

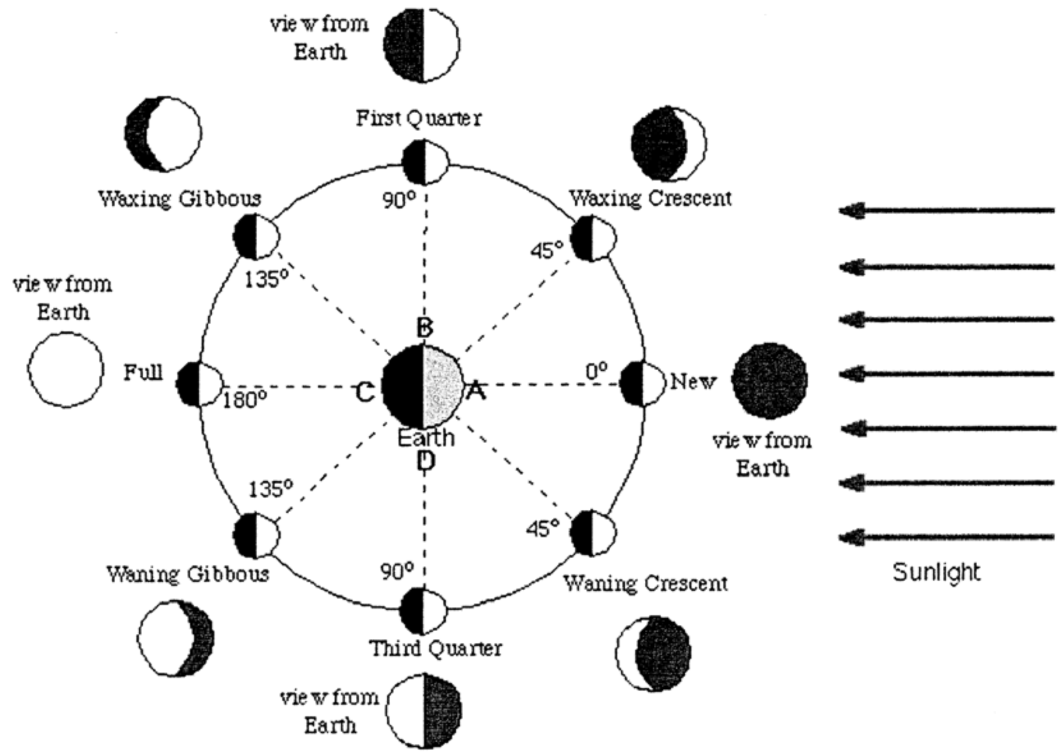
Earth's Rotation

NEAR THE END OF THE THIRTY-DAY OBSERVATION period, the Moon Unit begins a preliminary study of how Earth rotates on its axis while orbiting the Sun. A set of questions helps students determine that Earth is rotating eastward on its axis to cause day and night. An eastward rotation is counterclockwise, as seen from above the North Pole, which is the students' viewpoint when working with the globe. Each group of students stands with a globe in front of a flashlight or overhead projector; the bright light represents the illumination of the Sun. Following the guided directions and questions on **Student Page 1** (at the end of this article), students place a sticker on their hometown and turn the globe to explore Earth's rotation on its axis and how this rotation causes day and night. Students are required to work with the three-dimensional version while replicating Earth rotating on its axis as it orbits the Sun, then draw two-dimensional diagrams to illustrate what they saw.

The classroom teacher facilitates learning through questioning (especially when the directions tell them to check with the teacher), with such questions as these: What is the difference between day and night? Why are we not in total darkness all the time? In what direction does Earth rotate on its axis? How do you know? Can you show me how you know that the Sun rises in the east and sets in the west and, therefore, that Earth must be rotating eastward, or counterclockwise? Thus, as in mathematics classes, students communicate about their understandings and misunderstandings, revealing their thinking both to themselves and to their teacher.

Comparing Observation Data

AFTER COMPLETING THEIR ACTIVITIES WITH Earth, students begin working with the Moon. Students work with group members to create a Moon chart. They are given a blank calendar that has been outlined on a piece of poster board. Students work



The Sun-Moon angle is the angle defined by Sun→ Earth→ Moon with Earth (where *you are*) as the angle vertex. As the Sun-Moon angle increases, we see more of the sunlit part of the Moon. Note that if this drawing were to scale, then the Moon would be half this size and its orbit would be about 22 times larger in diameter and the Sun would be about 389 times farther away than the Moon!

Fig. 3 The phases of the Moon as seen from above and as seen from Earth

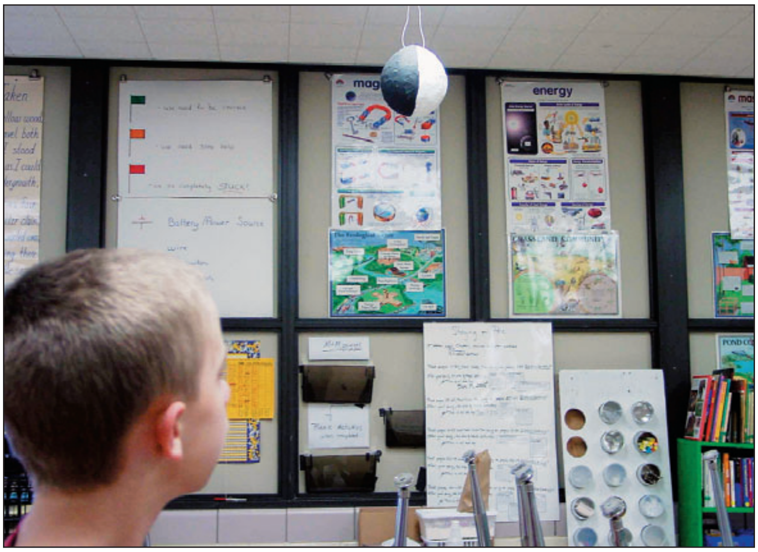
collaboratively and share their Moon journals with one another to compare the drawings of the shape of the Moon for each evening. This takes considerable time because students' data may differ, and they do not know which drawing to use in their Moon charts. They have now collected data, reconciled their observations with one another, and recorded it with a different type of representation—the Moon calendar. (See the photograph on the next page of a Moon calendar containing students' data.)

Phases of the Moon and Translating between Three-Dimensional and Two-Dimensional Models

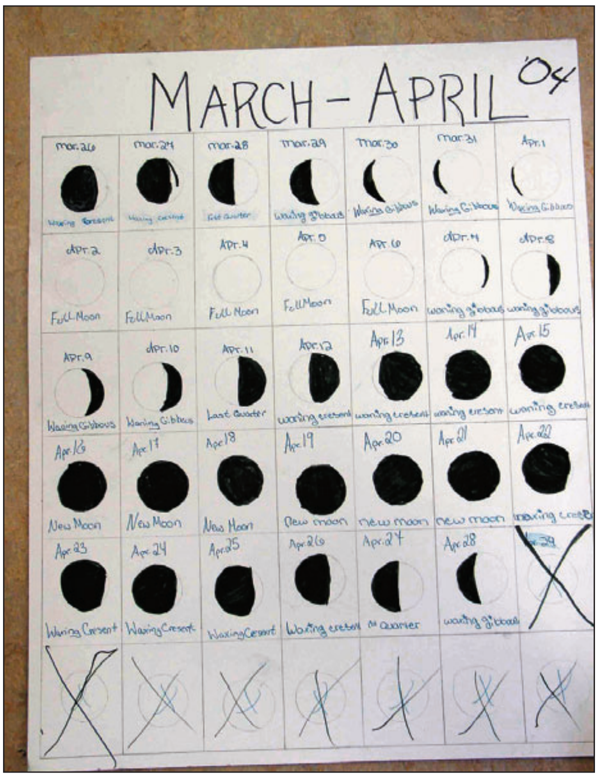
STUDENTS NEXT BEGIN WORKING WITH THE phases of the Moon. I give them a chart of the eight different phases of the Moon (see the Moon illustrations surrounding the diagram in **fig. 3**), and students must identify each phase that they have listed on their Moon chart. Here the students are observing the phases as viewed from Earth and studying two-dimensional representations of three-dimensional phenomena. When they are finished, students have a better understanding of the phases and will be able to recognize each phase when they see it again.

Once phases have all been named, students use the light from the overhead projector (the Sun) and a racquetball (the Moon) and note what they observe as the Moon revolves about the globe. The point of view used is as though they are seeing the Moon from Earth, following the directions and answering the questions on **Student Page 2** (at the end of the article). Students know from previous experiments that Earth rotates eastward, but they do not yet know how the Moon orbits Earth. Through inquiry, students experiment with the objects and eventually determine that the Moon is also traveling in a counterclockwise fashion around Earth. How do they do it? Most students use the Moon charts they developed and recognize the shape of the Moon from their observations. Using the Moon chart as a guide, students can see which phase should be next and then by manipulating the Moon, they are able to determine the direction the Moon is revolving. If they move the Moon the wrong way, the sequence of phases will be backward from their previous observations.

After completing their Moon-orbiting experiment, students must draw each phase of the Moon



Eight phases of papier-mâché Moons are hung in the classroom.



Students record data and produce a Moon calendar.

from both a bird's-eye view as well as their view from Earth. I have found this assignment to be difficult for students, because they do not understand the orientation of the objects representing the Moon from "above," as if a bird were seeing it. Students also have a hard time visualizing and connecting a bird's-eye view of a waxing crescent to an on-Earth view of a waxing crescent. To remedy this, I created all eight phases of papier-mâché Moons and hung them throughout my room from ceiling lights. The side painted white represented the side facing the Sun that reflects the Sun's light and, hence, is visible to an observer on Earth. When students are having difficulty understanding which phase goes with which drawing, I stand them on "Earth" in the center of the classroom and have them find the appropriate location of the Moon that correlates with the two-dimensional drawing. This really made a difference for my students' comprehension. **Figure 3** (Strobel 2004) is a diagram of the view from above and the view from Earth of the phases of the Moon and when they occur during the Moon's orbit.

After completing their phase drawings, students are quite familiar with the Moon and have a good understanding of how it looks during each phase. This is when I introduce the Moon clock to the students, a fun and easy project that helps students learn when each phase of the Moon is going to be visible in the sky. The Moon clock (currently available at www.lowell.edu/Public/Starlab/MoonClock.pdf#search='Moon%20clock') allows students to rotate Earth and watch a phase of their choice rise from the eastern horizon and set below the western horizon. Although this activity ultimately helps with understanding, one initial problem is that it shows more than one phase of the Moon at a time. For example, if we want to focus on a full Moon, students will see the full Moon rise at 6:00 p.m., but they will also "see" the waxing gibbous, the first quarter, and the waxing crescent, in addition to the full Moon. Students may become confused when they can see all the phases on the Moon clock and may need help focusing on just one phase at a time.

For the Moon clock to be successful, I allow students to play with it for a while and focus on working with Earth. They rotate Earth counterclockwise on its axis, and as the Moon comes up above the horizon, they see what time the pointer is facing to determine the time it rises. Although the moon rises at a different time each day, it always stays above the horizon for approximately twelve hours. In so doing, students are able to see the position of Earth in relationship to the Sun and the Moon. The point of view of the Moon clock is a bird's-eye view, reinforcing students' work transferring between three- and two-dimensional representations. Working with the Moon clock makes locating specific phases of the Moon at particular times much easier. Furthermore, by the end of the unit, students can determine when the Moon will rise, when it will be at its highest point in the sky, when it will set, and the times of the day during which they can observe a particular phase.

Eclipses

SOLAR AND LUNAR ECLIPSES COMPRISE THE NEXT segment of the unit. Earlier in the year, students learned about light and how it travels, reflects, absorbs, and bends and how shadows are created. One of the first questions they always ask is this: "Why don't we always have a solar eclipse during a new Moon and why don't we always have a lunar eclipse during a full Moon?" To answer this, they use a racquetball (the Moon), the globe, and the overhead projector (the Sun) to discover how it works. We discuss that a solar eclipse occurs when the Moon's shadow is cast on the Earth. Students create their own solar eclipse by moving the racquetball between the globe and the overhead projector. They soon realize that if the Moon were in the direct path between the Sun and Earth, we would have an eclipse every month. They know this does not happen. Thus, the Moon must not orbit Earth this way. The students also make a lunar eclipse by bringing the Moon in the shadow of Earth, seeing what is causing this to occur while realizing that a lunar eclipse does not happen every month. Students then draw two-dimensional representations to depict a lunar eclipse and solar eclipse, which they have just modeled in three dimensions. Eventually, students realize that the Moon revolves around Earth on a different plane than Earth's orbit around the Sun.

Teaching Strategies and Conclusions

THE FIRST FEW TIMES I TAUGHT AN INQUIRY LESson were overwhelming. I felt I was spread thin, constantly moving from group to group, sitting in, listening in, and helping out where needed. I eventually developed a three-flag process to avoid an overwhelming number of requests from eager students. A set of three flags—green, yellow, and red—are placed on each group's table and flags go up as needed by the group. Green indicates that the group is ready to be questioned about the material. Yellow indicates overall questions, and red means that the group is at a standstill. These flags allow the teacher to keep the entire class working by enabling needs to be addressed as they arise.

Not only is each section of the Moon Unit set up for student achievement and understanding but the process is a vital tool in developing responsible, self-motivated students. They begin to love and enjoy the responsibility of being accountable and on their own, since groups work at their own pace.

The inquiry process in science is analogous to constructing one's own understanding in mathematics through problem solving. Throughout the Moon Unit, students worked concretely with three-dimensional models to investigate Earth's counterclockwise rotation causing night and day and to explore the Moon's rotation and orbit about Earth. This is a natural setting for students to translate between three-dimensional models and their two-dimensional representations. The students developed their own units of measurement to approximate the measurement of the three-dimensional angle of the Moon's altitude, and they collected and analyzed their data to understand the Moon's phases. These mathematical tools helped the students understand the Moon's phases, the direction of the Moon's orbit, and the nonmonthly occurrence of eclipses. In turn, the science provided a natural context to develop and use mathematics concepts and tools.

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(*Worksheets are on the next page.*)

Student Page 1

NAME ________________________________

1. Explain how Earth is rotating, in your own words.

2. How do you know Earth is rotating in this direction?

3. Obtain a globe, a flashlight, and a smiley sticker. The flashlight represents the Sun in this model. Shine the flashlight at Earth. Draw a picture of the model you have just created.

4. Take the smiley sticker and place it on our hometown. Rotate Earth so that the smiley sticker is facing the Sun. What time of day is it if the smiley sticker is facing the Sun? Explain.

5. Draw a diagram of your model.

6. What time of day would it be on the other side of Earth? Explain and draw a diagram.

7. Continue rotating Earth so that the smiley sticker is no longer in the Sun. What time of day would it be now? Explain.

8. Draw a diagram of your model, then check with your teacher.

9. You have just created a model of Earth rotating on its axis. As you know, Earth rotates on its axis once every 24 hours. You have also created a day and night model. When we, represented by our smiley sticker, were facing the Sun, we had daylight. When the smiley sticker was not in the Sun, we had night time. How many hours of daylight do we usually have? Explain.

10. Now that we have determined why we have night and day, we will be working on figuring out the direction the Earth actually rotates on its axis. Using your globe and the overhead projector or flashlight again, place our hometown in the night. Draw a diagram of your model. Be sure to label where the sticker is.

11. Begin rotating the globe just until the sticker gets sunlight. What time of day is it once we begin seeing the sunlight over the horizon? Explain.

12. Continue rotating Earth on its axis just until the sticker is no longer in the sunlight. What time of day is it when the Sun no longer shines on the sticker? Explain.

13. Where did the Sun rise? How do you know?

14. Where did the Sun set? How do you know?

15. Draw a diagram of your model as if you were looking at it from above, like a bird's-eye view. Label the Sun, Earth, and the direction that Earth is turning.

16. In your own words, explain what is happening in the diagram.

Student Page 2

NAME ___________________________________

1. You will be modeling the phases of the Moon. Obtain a racquetball, which will represent the Moon; you will represent Earth; and the overhead projector will represent the Sun.

2. One student will stand in front of the overhead projector and hold the "Moon." The Moon, as you know, is illuminated by the Sun's rays that reflect off it and to our eyes. That is why you only see parts of the Moon lit up at times.

3. The student holding the Moon should begin by holding the Moon between the "Earth" and the "Sun." What phase of the Moon is this? Look at your chart if needed.

4. Now begin turning; you represent Earth. Remember, Earth rotates counterclockwise. Continue holding the Moon directly in front of you. As you turn counterclockwise, what do you begin noticing about the Moon?

5. Continue rotating and watch the Moon the entire time. You should have seen all the phases of the Moon.

6. Make sure each group member is able to be "Earth." Once everyone has been Earth, go back to your groups.

7. Draw each phase of the Moon that you saw. There are eight phases of the Moon. For each drawing, you will need a Sun, Earth, and Moon. You may also go back to your Moon charts to help you determine which phase you are looking at if needed.

8. Now draw each phase of the Moon from a bird's-eye view.