DEPARTMENT OF PHYSICS

LABORATORY MANUAL FOR PHYSICS 110

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Introduction to the Physics 110 Laboratory

Since the days of ancient Greece and Aristotle, physicists have tried to understand the nature of the world around us. Aristotle and his contemporaries speculated at great length about our universe; in fact, they devoted a great deal of their energy to constructing arguments about the nature of this universe. They did not, however, attempt to verify these arguments experimentally; thus, science and philosophy were indistinguishable. Hundreds of years later, physicists began performing experiments to try to test their ideas about the world (for example, Galileo's work with inclined planes). This methodology continued to develop, and evolved into the scientific method that forms the foundation of modern physics.

Experiment and observation are cornerstones of the scientific method. Whether gathered in a hands-on experiment or collected from a computer model, measurements generate the data that scientists use to test hypotheses and revise theories. As technology continues to advance, computers play an increasingly important role in physics. This semester, several of your labs will focus on the basic concepts of computational physics. The physics that underlie computational experiments and hands-on experiments is the same; that is, gravity is gravity, whether you drop a brick of a tower or model a rocket launch on a computer.

However, computational and hands-on experiments do generate different types of uncertainty. For example, in computational labs we usually try to model continuous functions with a set of discrete points. Consider the position of an object (x) as a function of time (t); if we graph x versus t, we get a continuous graph. On the other hand, if we use a computer to model the same situation, we will be left with a collection of ordered pairs (x,t) which are not continuous. This kind of approximation can lead to truncation error (where the computer's estimate of a number differs from the true value of a number because the computer has to chop off some of the end of the number) and round off error (where the error arises from the computer rounding numbers). Hands-on experiments have different types of error; usually, this is because hands-on experiments are more complicated and involve more steps then computer simulations. The remainder of this introduction will focus on uncertainties in hands-on experiments; you will discuss uncertainties in computational experiments in greater depth during lecture and the computational labs.

In physics (and in science in general), a measurement has two numbers (e.g., $h=14 \pm 3$ cm). The first number is the best estimate of the true value of the quantity being measured (often called the central value), and the second number is an estimate of the uncertainty in the measurement. Unless we are counting discrete objects (for example, how many apples are in the barrel), uncertainty is always present in a measurement. Error occurs for a variety of reasons. For example, instruments may be calibrated incorrectly, environmental conditions may not be properly controlled, or there may be uncertainty involved in reading the instrument. Because we cannot avoid error, we must do our best to quantify it; this helps to ensure that we may draw accurate conclusions from our experiments. You should use the definitions below to become familiar with different types of error and uncertainty; they are essential to understanding work that takes place in laboratories.

Definitions

Error: The difference between a measured value and the true value. Usually, the true value (or exact value) is unknown, so we must estimate the error.

Discrepancy: The difference between two measurements. For example, suppose you and you lab partner both measure the length of a rod. If you say that the rod is 1.2 meters long, and your partner says the rod is 1.4 meters long, then the discrepancy between the measurements is .2 meters (Notice that we are only looking at the central values in this example).

Uncertainty: Uncertainty is the estimated maximum error in a measurement or calculated quantity. We express the uncertainty as a range; we use the range to set limits for the possible value of our measurement. For example, suppose that we measure a rod and record its length as L = (12.5 ± .8) cm. The first number (12.5 cm – the central value) is our estimate of the length of a rod, and the second number (.8 cm) is the uncertainty in the length. Notice that we are claiming that the length of the rod falls somewhere between 11.7 cm and 13.3 cm.

Absolute Uncertainty: The absolute uncertainty of a measurement is the (actual) value of our error estimate. In the preceding example, the absolute error is ± .8 cm.

Relative Uncertainty: The relative uncertainty of a measurement is the ratio of the absolute uncertainty to the measured value (we usually express this ratio as a percentage). In the example above, the relative uncertainty is: $\pm\frac{\text{absolute uncertainty}}{\text{measured value}} = \pm\frac{.8\text{cm}}{12.5\text{cm}} \text{x}100\% = \pm 6.4\%$.

Random Error: As the name suggests, a random error has an equal chance of being larger or smaller than the measured value; therefore, random error tends to "cancel out" over a large number of measurements.

Systematic Error: A systematic error is error that tends to be either consistently larger or smaller than the measured value. For example, if a scale is not zeroed before we weigh several objects, we might record values that were several grams less than the actual weight of the objects. Because systematic error occurs in only one direction, it can distort the results of experiments significantly.

Blunder: A blunder is an error made by an experimenter because of faulty observations or incorrect computations. For example, the experimenter who uses the value $\pi = 31.4$ has made a blunder in his or her calculation.

Significant Digits: The significant digits in a measurement are the number of digits required to accurately report a measurement. For example, our previous length measurement – L = (12.5± .8) cm has three significant digits (that is, the central value has three significant digits).

Propagated Error: Propagated error is error that accumulates through several (related) calculations. For example, when you calculate the volume of a right circular cylinder ($V = \frac{\pi}{4}D^2\ell$, where D is the diameter of the base and ℓ is the length of the cylinder) you might measure D and ℓ. In order to determine the error for the volume of the cylinder, you must use the error associated with the height measurement (ℓ) and the error associated with the diameter measurement (D). In other words, you must propagate your error.

Accuracy: Accuracy is a measure of how closely a measured value agrees with the true value. High accuracy in a measurement means that there is a small amount of error.

Precision: Precision is a measure of how closely a set of individual measurements (of the same quantity) agree with one another. High precision implies a small spread of individual measurements about the mean.

Estimation of Uncertainties

Usually, when we measure a quantity, we do not know its true value; therefore, we do not know the true error associated with our measurement. Ideally, we want to find the best estimate of the true value, and then estimate the error. We usually call this error estimate the uncertainty in our measurement. Detailed error analysis can be time-consuming; often the analysis takes longer than the experiment. In this course, we will use a simplified procedure to estimate uncertainties; as your understanding of physics and statistics progresses, you will learn techniques that will allow you to refine the methods that you learn now. Finally, note that you will find that it is easier to try to minimize error in the experiment by working carefully throughout the experiment instead of trying to make corrections after the fact.

We will begin by considering a single measurement, x, of some quantity whose true value is x_t. Then the uncertainty (Δx) in our measurement is related by the equation

$$x = x_t + \Delta x$$

In general, each measurement will involve a different amount of error. If we assume that the error is predominately random error, we can minimize their effect by taking repeated observations and averaging the results. This method yields an average measurement of

$$\overline{x} = x_t + \frac{\Delta x_1 + \Delta x_2 + ... + \Delta x_N}{N} = x_t + \frac{\sum_i \Delta x_i}{N}$$

for a set of N measurements (notice that the summation sign means that we are adding up all of the Δx's – the i's just remind us that each Δx is (in general) different). Because of the algebraic signs of the Δx_i, the difference between $\overline{x}$ and x_t should be much smaller than the difference between individual x's and x_t. Unfortunately, this procedure does not work when we are dealing with systematic errors, because systematic errors always have the same sign. We can reduce systematic errors by working carefully, paying special attention to the calibration of instruments, and correcting for zero settings of measuring devices. We can eliminate blunders by careful work; therefore, blunders should not be cited as a source of error. If you make a blunder, redo the affected part of the experiment! The remainder of our discussion assumes that careful procedures have minimized the systematic error, and that most of our error is random.

We will assume that the arithmetic mean is the best estimate of the true value of any quantity under observation (that is, we should use the mean as the central value). Suppose that we measure a quantity x a total of N different times, then the mean (or average) is given by

$$\bar{x} = \frac{\sum_i x_i}{N}.$$

We can estimate the precision of our measurement by examining the spread of the individual measurements about (around) the mean. If the systematic error is negligible, then the mean of our measurements is close to the true value of the quantity. Furthermore, the uncertainty in the measurement is related to the spread of the data. Two common measures of precision are the mean deviation and the standard deviation. These are defined by

$$\text{mean deviation} \quad d_x = \frac{\sum_i \left| x_i - \bar{x} \right|}{N} = \frac{\sum_i \left| \Delta x_i \right|}{N}$$

$$\text{standard deviation} \quad \sigma_x = \sqrt{\frac{\sum_i \left(x_i - \bar{x} \right)^2}{N - 1}}$$

Both of these measures may be used to estimate the spread of the data. Notice that the mean deviation is similar to the expression for the error associated with a measurement that we presented at the beginning of the section. In fact, the only difference is that when we use the mean deviation, we sum the absolute values of the error, and not just the error itself – we are considering the worst case scenario where none of the error cancels out. Usually, experimenters will use the mean deviation when N<20. However, because many calculators can calculate the standard deviation, we will always use the standard deviation to estimate our error. Also bear in mind that estimates of error are only estimates; hence, we will only use one significant figure when we report our uncertainties.

Obviously, if we only make one measurement, neither the mean nor the standard deviation are useful in estimating the error. Therefore, when we make a single measurement, we will use the limits of our measuring device to estimate our error. We will call the closest measurement that we can make with an instrument the least measure of the instrument. For example, a meter stick is typically divided by hash marks that are one millimeter apart; hence, the least measure of the meter stick is one millimeter. When we measure something with a meter stick, however, we usually try to guess one "extra" digit. So we will usually record our result to one tenth of a millimeter (e.g., 1.4 mm). We will also use this convention with other measuring devices whenever it is practical (that is, we try to guess an extra digit). Finally, note that we will usually take the error of a (single) measurement to be one half of the least measure. So for a typical meter stick with a least measure of one millimeter, we would record the error of a (single) measurement as one half millimeter.

Significant Digits

Consider the measurement x = (2.07 ± 0.2) m. Notice that the third digit in the measurement (7) is meaningless. Without the error (if we reported our measurement as x = 2.07 cm), the result would be correct to three digits, and we would expect the value to be somewhere between 2.02 and 2.12. But the ± 0.2 encompasses (and expands) this range, so we could just as easily record our result as x = (2.1 ± 0.2) m. In fact, the latter answer is a more accurate description of the range of the data. In general, you should report your error with this method. So, if you measure something and then determine that the error is ± 0.1 m, you should report your answer to the first digit after the decimal place (15.6 m, for example). Also remember to take significant figures into account whenever you present (or interpret) numbers that could include error. For example, the figure x = 600 mm is ambiguous; we do not know if the zeros in the measurement represent significant figures or not. Suppose that the first zero in the measurement is significant, and the second one is not, then the correct way to present the measurement is $x = 6.0 \times 10^2$ mm.

We must also consider significant figures when we perform arithmetic (that is, when we add, subtract, multiply, or divide numbers with significant figures in them). We will consider multiplication and division first. When we multiply (or divide) two numbers, we might end up with more significant figures than we started with. We have a rule that helps us avoid this problem; whenever we multiply or divide two (or more) numbers, we only keep as many significant figures as are in the number with the least number of significant figures. For example, $32.457 \times 12 = 3.9 \times 10^2$, where we only keep two significant figures because the second number has only two significant figures. For another example, note that 39 x .342 = 13; in this case, the first number limits the number of significant figures in our answer. Because we can consider division as multiplication of the inverse of the denominator and the numerator, we may treat division the same way that we treat multiplication. Next we will examine addition (remember that we can treat subtraction as the addition of a negative number, so we treat subtraction the same way that we treat addition). When we add two numbers, we determine the number of significant figures from the number having significant figures in the largest positions. Several examples presented below should clarify:

$$\begin{array}{r} 32.?? \\ +\ \ 4.21 \\ \hline 36.?? \end{array} \qquad \begin{array}{r} 469.832 \\ -\ \ \ 6.1?? \\ \hline 463.7?? \end{array} \qquad \begin{array}{r} 2978 \\ +\ 3?? \\ \hline 32?? \end{array}$$

Note that the answer to the last example should be expressed as 3.2×10^3.

Propagation of Uncertainties

If R is a function of x and y, then we may write R = R(x,y). If x and y are independent of one another, then the error in R caused by random error in x and y may be approximated by

$$\Delta R = \left|\frac{\partial R}{\partial x}\Delta x\right| + \left|\frac{\partial R}{\partial y}\Delta y\right|.$$

We use the absolute value bars to ensure that the value of ΔR is as large as possible[*] (given the two quantities on the right hand side of the equation), this assumes the worst case scenario, that the sign of the error in x and y have the same sign. We will find two formulas involving special cases of this error particularly helpful throughout the semester; in words, the formulas say that when we add or subtract numbers, we should add the absolute error associated with the measurements; on the other hand, when we multiply or divide numbers, we should add the relative uncertainties associated with the measurements. Also, when we multiply numbers with exponents, we should also multiply the exponent by the relative error in the measurement. In symbols, the formulas say:

$$\text{When} \quad R = x + y,\ \Delta R = \Delta x + \Delta y. \qquad (1)$$

$$\text{When} \quad R = x^n y^m,\ \frac{\Delta R}{R} = n\frac{\Delta x}{x} + m\frac{\Delta y}{y}. \qquad (2)$$

Note that in (2), n and m may be positive or negative, so the formula also covers division. As an example of (1), consider R = x-y = (24.5 ± 0.4) – (11.3 ± 0.3) = (13.2 ± 0.7). As an example of (2), consider $R = \frac{x}{y} = \frac{(2.0 \pm 0.2)}{(5.1 \pm 0.3)} = \frac{(2.0 \pm 10\%)}{(5.1 \pm 6\%)} = 0.392 \pm 16\% = 0.39 \pm 0.06$. Note that we divided central values of the numerator and the denominator to get the central value for R and we added the relative uncertainties of the numerator and the denominator to get the relative uncertainty for R from the rules. While these examples (and some of the other things that you have encountered in the introduction) might seem intimidating and confusing, rest assured that with some practice, you will become adept at propagating error, and most of the concepts presented here will become second nature. As you complete the labs this semester, bear in mind that you cannot learn physics without doing physics. If you approach the laboratory as a chance to develop your physical intuition, you should find the labs rewarding and enjoyable.

Exercises

The next two pages contain exercises that will help you become familiar with the concepts we discussed in the introduction. Part I is for practice. You need to turn in the problems in Part II.

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[*] For those of you who have had calculus, note that $\frac{\partial R}{\partial x}$ is called a partial derivative. Locally, we find $\frac{\partial R}{\partial x}$ by holding y constant and taking the derivative of R with respect to x; a similar process yields $\frac{\partial R}{\partial y}$.

Exercises on Significant Figures and Uncertainties

I. Practice Problems

1) Write the number of significant digits for each of the following numbers.

5.00 ________ 4000.6 ________

0.00275 ________ 2.64×10^3 ________

3.0×10^5 ________ 0.0006 ________

2) Write the number of significant figures that should be in the answers of each of the following calculations.

$\frac{34280 \times 265}{5347}$ ______ $\frac{65.432}{15}$ ______ $\frac{3425}{2.0}$ _______

$\frac{3425}{2.000}$ ______ $\frac{25}{3.1416}$ ______ 475^2 _______

$\frac{2.3 \times 10^4}{5362}$ ______ $3.54 \times \pi$ _____

3) Answer each of the following questions, remembering to pay attention to significant figures and error propagation.

(23.088 ± 0.004) mm + (15.843 ± 0.002) mm = ___________________________

(23.088 ± 0.004) mm + (15.84 ± 0.04) mm = ___________________________

(1.33 ± 0.02) x (2.52 ± 0.03) = ______________________________

(1.33±0.02) x (2.523 ± 0.005) = _____________________________

4) For each of the following sets of data find $\bar{x} \pm \sigma$

	Set a	Set b	Set c
x_1	1.59 cm	23.093 cm	2.53 cm
x_2	1.59	23.103	2.75
x_3	1.60	23.103	2.33
x_4	1.59	23.094	2.15
x_5	1.60	23.098	2.41
x_6	1.60	23.097	1.99
Least meas.	0.01 cm	0.001 cm	0.01 cm

(a) ___________±_______

(b) ___________±_______

(c) ___________±_______

Exercises on Significant Figures and Uncertainties

II. Hand In Problems

1) Write the number of significant digits for each of the following numbers.

$1.23\text{x}10^{-5}$ ________

320.2 ________

0.06 ________

2.10 ________

0.00392 ________

$0.33\text{x}10^{2}$ ________

2) Write the number of significant figures that should be in the answers of each of the following calculations.

$\dfrac{3.33 \text{ x } 10^{-5}}{5.186 \text{ x } 10^{-8}}$ ______

$\dfrac{1.002 \text{ x } 2553}{35921}$ ______

512^{2} _______

$5.12689\text{x}10^{5} \text{ x } \pi$ ______

0.25 x 333 ______

$\dfrac{3\pi}{2.5}$ _______

3) Answer each of the following questions, remembering to pay attention to significant figures and error propagation.

(13.020 ± 0.012) cm + (35.312 ± 0.009) cm = ___________________________

(1.352 ± 0.002) cm x (36.814± .009) cm = ______________________________

4) Find the volume of a cylinder of diameter (15.843 ± 0.002) mm and a length of (23.088 ± 0.004) mm.

V = _______________

5) For each of the following sets of data find $\bar{x} \pm \sigma$

x	Set a	Set b	Set c
1	2.32 cm	1.9272 cm	2.53 cm
2	2.32	1.9312	2.42
3	2.32	1.9325	2.39
4	2.32	1.9301	2.45
5	2.32	1.9263	2.55
Least meas.	0.01 cm	0.0002 cm	0.01 cm

(a) ___________±_______

(b) ___________±_______

(c) ___________±_______

Experiment L-1: Visualizing Functions by Tracing a Baseball

Purpose: During this experiment, you will use a computer, a mathematical model, and several graphs to visualize and analyze the path of a projectile. You will also have the opportunity to familiarize yourself with a programming language (fortran) and several departmental computer resources (e.g., entropy).

Introduction: One of the key concepts in physics is the idea of studying the world around us with mathematical formulas. During the last several weeks of class, you have derived and studied several of the formulas of motion. Today, we will explore the flight of a baseball using some of these formulas. Specifically, we will use the formulas that tell us the horizontal and vertical position of a baseball (or any projectile, for that matter) as a function of time. If we call the horizontal position x, the vertical position y, and the time t, then our equations become:

$$x(t) = v_0\cos(\theta_0)t \qquad (1)$$

$$y(t) = v_0\sin(\theta_0)t - \frac{1}{2}gt^2 \qquad (2)$$

Where v_0 is the initial velocity of our baseball, θ_0 is the initial angle of our baseball (relative to the x-axis), and g is the acceleration caused by gravity.

Using these formulas, we can calculate where the ball will be at any time t (given v_0 and θ_0, which are called the initial conditions). If we like, we could construct a table of these values, so that we could look up where the ball would be at some time t. In fact, we could construct several tables to find out where the ball would be if we change v_0 and θ_0. This would allow us to compare what happens to the ball for different initial conditions. Unfortunately, even for relatively simple functions like these, generating more than one or two tables becomes very time consuming. It is also difficult to analyze what is happening to the ball based on a bunch of tables. These problems are even more troublesome for more complicated functions (the position of the space shuttle during a launch, for example).

So what we would like is a way to generate these tables quickly and efficiently, and an effective method for analyzing the tables. Luckily for us, computers are good at generating tables of data, and the physics department provides a program that turns tables of data into graphs. So all we need to do is tell the computer to make us some tables, and then tell the graphing program (Kaleidagraph) to make us some graphs.

Unfortunately, computers aren't quite smart enough to do all of this on their own (yet…). We will have to be careful when we make our instructions; we need to design them so that the computer understands what to do. With this in mind, we will create and use a fortran program to evaluate our functions and generate data tables. Next, because the fortran compiler (and your program) is located on entropy, and Kaleidagraph is located on the computer that you are using, we will use an FTP (which stands for File Transfer Protocol) application (Fetch) to move your data file from entropy to your computer. Finally, we will use Kaleidagraph to turn our data tables into graphs.

Procedure:[*]

Part I – Making and Running a Fortran Program

The first thing that you should do is to make sure that your computer is on, and that the keyboard and mouse are working properly. Next, open up the telnet application and log on to entropy. Once you are logged on to entropy, you should create a fortran program that calculates x(t) and y(t) for discrete values of t. Your program should evaluate the functions at 21 (equally spaced) points starting at t = 0. Take $\Delta t = .328$ seconds (Note that we will take x(t) and y(t) to be in meters). Your program should also output the results to a data file (our table). Your data file should have three columns of data (corresponding to t, x, and y) with 21 entries in each column. For this part of the experiment, take $v_0 = 50\,\frac{\text{m}}{\text{s}}$ and $\theta_0 = 40^\circ = \frac{40\pi}{180}$ radians. Also remember that the sine and cosine functions are intrinsic to fortran (that is, fortran knows how to compute them), but π is not (so you must give the computer a value for π; try pi = 4.*atan(1.), where atan means the inverse tangent function). Compile your program, debug it, and then run it. Print out a copy of your data file and your source code, and turn them in with your report.

Part II – Graphing Data with Kaleidagraph

Next, use Fetch to move your data file from entropy to the computer that you are using. Open the data file using Kaleidagraph, and make graphs of x versus t and y versus t. (make sure that the data points are visible as data markers and do not “connect the dots”). Print out the two graphs and turn them in with your report.

Part III – New Initial Conditions

Repeat Part I using the initial conditions $v_0 = 50\,\frac{\text{m}}{\text{s}}$ and $\theta_0 = 65^\circ = \frac{65\pi}{180}$ radians. This time, make $\Delta t = .462$ seconds, and calculate t at 21 points. You don’t have to print out the source code for this program, but please print out the data file and include it in your report. Next, bring your data file to your computer, and use Kaleidagraph to graph y versus x for this set of data (notice that this is a graph of the trajectory of the baseball). On the same graph, plot y versus x using the data from the first graph. Print out the graph and remember to turn it in with your report.

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[*] Please note that we will not be discussing the specifics of how to write a fortran program, use Fetch, navigate UNIX, or make graphs using Kaleidagraph in this manual. The class notes – as well as the department’s computer manual – contain detailed instructions on these topics; your T. A. should be able to answer any questions that the manuals don’t.

Questions:

1) Using the graphs of x(t) and y(t) (from Part II), determine when the baseball reached its maximum height. Also determine the height and horizontal run (displacement) at that point. Be sure to explain how you got your answers.

2) Using the same graphs, find out when the baseball hit the ground (you may assume that the ground is flat and that there are no obstacles for the ball to run into). What is the total range of the baseball? Again, remember to explain how you got your answers.

3) Using the graph of the baseballs' trajectory (from Part III), find out the maximum height and horizontal range of both of the baseballs. Compare this data to the data from the first two graphs.

Experiment L-2: Velocity of a Mass-Spring System – Numeric Differentiation

Purpose: In this lab, you will use a computer to model a mass attached to the end of a spring. In the process, you will learn how to use a computer to estimate derivatives.

Introduction: One of the most basic applications of derivatives involves defining velocity to be the derivative of displacement with respect to time (for one dimension, $v = \frac{dx}{dt}$). From time to time, we would like to have a way to tell computers to take these derivatives for us. Unfortunately, most programming languages (like fortran and C) cannot perform the abstract mathematics needed to evaluate derivatives directly. While it is possible to write a program that contains all of the rules for differentiating functions, it would be extremely complicated to code, much less use.

What we would like is a "quick and dirty" method for computing numerical derivatives that we can use on a wide variety of functions. Luckily, reformulating the definition of velocity (and of the derivative in general) allows us to compute many derivatives relatively easily. If we define velocity to be the limit of an average velocity as the change in time for the interval in question approaches zero (which is a valid definition), then all we have to do is tell the computer to calculate several average velocities for decreasing time intervals. Then, we look at the behavior of the average velocity as the time interval gets smaller and smaller; we should be able to deduce the derivative from this information (in other words, we want the computer to calculate several difference quotients – we will inspect the difference quotients and deduce the derivative).

Now that we have a method that a computer should be good at, we need to worry about the specifics of our program – how do we go about computing average velocities. Consider some function x(t); for this function, the definition of average velocity is: $\overline{v} = \frac{\Delta x}{\Delta t}$. We may reformulate the numerator as $\Delta x = x(t + \Delta t) - x(t)$; now we have a way to compute $\overline{v}$ in terms of x, t, and Δt (which is what we want). Now we may vary Δt as much as we like and compute various values of v.

For this experiment, we will consider a mass attached to the end of a spring. Neglecting friction, the position as a function of time may be expressed as $x(t) = \cos(t)$, in meters. To determine the instantaneous velocity, we will examine several average velocities as Δt goes to zero. We will vary the change in time using the following formula: $\Delta t = \frac{1}{2^{(i-1)}}$ for $i = 1,2,\ldots,15$, in seconds; while this formula is somewhat arbitrary, it does allow us to sequentially reduce our value for Δt.

Procedure:

Part I – Making the Fortran Program

Write a fortran program that computes Δt, Δx, and $\bar{v} = \frac{\Delta x}{\Delta t}$. In this part of the experiment, use a value of $t = 0$, and use 15 steps (that is, make i run from 1 to 15). Make sure that your program creates a data file with columns for Δt, Δx, and $\bar{v}$. Run your program. Make sure that you print your program and the data file.

Part II – Preparing the Data for Analysis

Move the data file from entropy to your desktop. Using Kaleidagraph, make a plot of $\bar{v} = (\Delta x/\Delta t)$ as a function of Δt; you will want to use a logarithmic horizontal scale. Next, make a second graph, and plot $x(t)$ as a function of t from $t = 0$ to $t = 3.5$. Print two copies of this graph. You may modify your program to generate data for the graph, or you may use Kaleidagraph's formula entry feature.

Part III – New Conditions

Modify your fortran program by setting $t = \pi/2 = 1.5708$ s. Rerun the program (using 15 steps, as before), bring your data file to your desktop, and make a plot of $\bar{v}$ as a function of Δt (again, you should use a logarithmic horizontal axis). Be sure to print out the data file, but do not worry about printing the source code this time (make sure that you turn in one copy of the source code, though).

Questions:

1) Consider the graph of $\bar{v}$ vs. Δt for $t = 0$. From the graph, what is the extrapolated value of $\bar{v} = \frac{\Delta x}{\Delta t}$ as $\Delta t \to 0$? (This limit is the numerical derivative dx/dt at $t = 0$).

2) Using one of the copies of your graph of $x(t)$ vs. t, draw lines whose slope corresponds to $\Delta t = 1, 0.5, 0.25$, and 0. You should have 3 secant lines and one tangent line, all passing through the point at $t = 0$. Using this graph and the printed data, what is the average velocity ($\bar{v}$) for each time interval?

3) Redo questions (1) and (2) using the data and graphs for $t = \pi/2$. Remember that we are still interested in finding out what happens when $\Delta t \to 0$, it is t that we are changing. This time, your secant lines should all pass through the point at $t = \pi/2$.

4) Comment on the advantages and disadvantages of numerical derivatives (as compared to symbolic derivatives).

Experiment L-3: Where is Pooh – Numeric Integration

Purpose: In this lab, you will learn a numerical technique that will allow you to integrate many functions using a fortran program.

Introduction: Suppose that Winnie the Pooh is falling through honey in a honey jar. As Pooh falls, imagine that Christopher Robin is watching from the outside of the jar, and recording Pooh's velocity during the descent. We are interested in finding the depth that Pooh has fallen (in other words, the vertical distance) as a function of time.

Suppose that Pooh is falling at a constant rate (that is, his velocity does not change). Then our question becomes trivial, and we can use the formula distance = rate * time. Pooh's velocity is not constant, however, so we need a better way to calculate the distance that he has fallen (his depth). We find this better way by imagining that Pooh's trip is actually a collection of very short trips. While Pooh's velocity may change by a great deal over the course of his entire trip, it does not change very much over the first 0.1 seconds. So we can take Pooh's velocity at the beginning of his trip and estimate how far he has traveled in the first 0.1 seconds. Next, we check Pooh's velocity at t = 0.1 seconds, and use that to calculate how far he travels in the next 0.1 seconds (from t = 0.1 seconds to t = 0.2 seconds). We continue adding these small pieces of Pooh's trip until we have an estimate of how far he traveled over his entire trip (those of you who have had calculus may notice that we have constructed a Riemann Sum). If we decide that our estimate of Pooh's distance is not good enough, we may shrink the interval that we assume Pooh's velocity to be constant over (0.1 seconds in our example – also called the time step) until our estimate is good enough.

Next, we would like to quantify our discussion from the previous paragraph so that we can tell the computer how to do the work for us. Suppose that Pooh's velocity is given by some function v(t), we use a time step of Δt seconds, and we assign his displacement to be in the x direction. Then the distance of Pooh's first little trip is given by $x_1 = v(\Delta t)*\Delta t$; his second trip is given by $x_2 = v(2\Delta t)*\Delta t$, and so on. The total distance that Pooh travels after the second interval is given by $x_t = x_1 + x_2$, after n intervals, the total distance is $x_t = x_1 + x_2 + \ldots + x_n$. In terms that a computer can understand, we want to use the formula $x = x + v*\Delta t$, where we add the contribution of Pooh's latest trip to the total distance each time we recalculate his velocity. The method that we have developed and will use for this experiment is a summation (that is, we are adding up a bunch of stuff), and it approximates the integral given by

$$x(t) = \lim_{\Delta t \to 0} \sum_i v(t_i)\Delta t = \int v(t)dt$$

In your program, you will find that you need to use a do loop to estimate this sum.

Procedure:

Part I – Making the Fortran Program

Assume that Pooh's velocity is given by $v(t) = 1.5(1 - e^{-t})$ and that $\Delta t = 0.1$ seconds. Write a fortran program that estimates Pooh's position over 6 seconds. When you output your data to a file, make sure that you include columns for t, v, and x. Print out your source code and your data file to turn in with your final report.

Part II – Creating Graphs

Using Kaleidagraph, graph v(t) and x(t) as functions of time (on separate graphs). Hand in a copy of both graphs with your report.

Questions:

1) Examine the graph of v(t) as a function of time, and use it to find Pooh's terminal velocity (that is, the limiting velocity that he reaches).

2) Examine the graph of x(t) as a function of time (this is the numerical integral of the velocity graph). Comment on the shape of the x(t) curve as Pooh reaches his terminal velocity. What is the slope of the x(t) curve for large t?

3) In the previous lab, you wrote a fortran program designed to numerically differentiate a function, and in this lab, you wrote a program to numerically integrate a function. Find and comment on at least one similarity and one difference in the two labs.

Extra Credit (5 points):

Modify your fortran program so that it computes the area of a quarter circle whose equation is $v(t) = \sqrt{1 - t^2}$ (for $0 < t < 1$). Multiply the area you obtain by a factor of 4 and you should get an estimation of the value of π. Using Nt = 10, 40, and 80, estimate π and compute the percentage difference between your estimation and the true value of π. Hand in your source code and the data set for 10 intervals. Make sure to report your results for the other data sets.

Experiment L-4: Falling Apple – Numerical Solution of Newton's Equation

Purpose: In this lab, you will model a differential equation using a fortran program.

Introduction: Before we consider the model that we will use for our experiment (a falling apple), we should briefly discuss the nature of differential equations. Unfortunately, many students have an irrational fear of differential equations during the course of their initial calculus sequence. Many of these fears are unfounded; a differential equation is just an equation that uses information about the rate of change of a given function in addition to the function itself (derivatives, in other words). Admittedly, solving differential equations analytically can be difficult and even impossible; fortunately, we are only interested in developing a technique that will allow us to estimate solutions to certain types of differential equations numerically.

In order to develop our method (which is known as the Euler method), consider the following situation. Suppose that an apple is falling towards the ground. Neglecting air resistance, we would like to use differential equations to determine the position (and velocity) of the apple at some time t after it begins to fall. We will assume that our apple is always close to the surface of the earth so that we may assume gravity is a constant 9.8 m/s^2; we will also stipulate that we know the initial position (x_0) and the initial velocity (v_0) of the apple. Notice that you should be able to determine the position and velocity of the apple without resorting to differential equations; this method is used here in order to illustrate Euler's method.

Before we start writing fortran code, we need to express what we know about this situation in terms of differential equations, and then rewrite our equations so that we may use them in a fortran program. In general, Newton's Second Law tells us that F = m*a; rewriting this equation, and using the fact that a = dv/dt and the definition of velocity, we have

$$\frac{dv}{dt} = \frac{F}{m}, \qquad \text{and} \qquad \frac{dx}{dt} = v.$$

These are the differential equations that we will use to estimate the position and velocity of the apple. For a small enough time step, we may estimate dv/dt and dx/dt in the above equations by

$$\frac{v(t + \Delta t) - v(t)}{\Delta t} = \frac{F(t)}{m} \quad \text{and} \quad \frac{x(t + \Delta t) - x(t)}{\Delta t} = v(t).$$

Notice that for small enough Δt, these equations are reasonable approximations of our original equations. Next, we need to rewrite the equations in terms that a fortran program can understand. The following equations may be used as fortran commands; you should convince yourself that they are equivalent to the equations that we just wrote down.

$$v = v + \frac{F}{m} * \Delta t \qquad (1)$$

$$x = x + v * \Delta t \qquad (2)$$

These two equations are the heart of the Euler method. Note that equation (2) is really a shorthand way of writing two equations:

$$x_{new} = x_{old} + v * \Delta t \qquad (2a)$$

[write the value of x_{new} to a file]

$$x_{old} = x_{new} \qquad (2b)$$

Therefore, for the original equation (2), we have to remember to write the value of x to a file before we execute the command. In our assumptions, we said that we wanted to know the initial velocity and the initial position of the apple. Also remember that F/m represents the gravitational force on the apple (so F/m = g). In order to calculate the position of the apple at time Δt, we use the initial values of x_0 and v_0 in equation (2) to calculate x. We may also use equation (1) along with the initial velocity to calculate the velocity at time Δt. Notice that we just calculated v(Δt) and x(Δt); in order to calculate v(2Δt) and x(2Δt), we substitute v(Δt) for v_0 and x(Δt) for x_0 in equations (1) and (2). We may continue in this manner for as long as we please, using the results of the previous step to calculate the current values of x and v.

To recount our strategy, we will use our initial conditions to approximate the velocity and position of our apple after a (small) amount of time has passed. We will then use these approximations to estimate the velocity and position after another amount of time has passed. We may (in theory) continue this method indefinitely; thus, we may estimate the position and velocity of our apple for any time t. In reality, however, we must pay close attention to the error associated with the Euler method. In fact, any error that we introduce into the experiment (by making approximations) will be magnified as the experiment progresses (why?). Also bear in mind that the larger the time step, the worse the approximation, and the larger the error.

Procedure:

Part I – Making the Fortran Program

Create a fortran program that uses differential equations to model a falling apple. You may use initial conditions $x_0 = 0$ m and $v_0 = 0$ m/s. Have your program send the following seven columns of data to your data file: time (t), displacement (x), theoretical displacement ($x_{theoretical}$), $\delta x = |x - x_{theoretical}|$, velocity (v), theoretical velocity ($v_{theoretical}$), and $\delta v = |v - v_{theoretical}|$. In this context, x and v are the numerical solutions to our differential equations, and the theoretical values are the exact solutions. In the program, use Δt = 0.1 seconds, and run the program from t = 0 to t = 10 seconds. Print out the source code and the data file and include them in your report.

Part II – Creating Graphs

Create a graph of x(t). In your graph, make sure to graph both the theoretical solution and the exact solution, using different markers for the two lines. Next, repeat the procedure for v(t), making sure to use different markers for the theoretical solution and the exact solution.

Questions:

1) How good are the numerical solutions for x and v?
2) Why does increasing the time step in the experiment increase the error?
3) If we decrease the time step, we decrease the error associated with the experiment; is it possible to use a time step small enough to eliminate the error in this experiment? Why or why not?
4) Suppose that we decide to include air resistance in our analysis. Describe what would happen to your values for x and v?

Experiment L-5: Measurement, Error, and Uncertainty

Purpose: The purpose of this experiment is to become familiar with some of the techniques of measurement and data analysis.

Introduction: The history of the irrational number π (where π is the ratio of the circumference of a circle to the diameter of the circle) is both long and interesting. The ancient Greeks originally noted the significance of π, and over the centuries many people have invested a great deal of time estimating the value of π (currently, computers allow us to estimate π to many digits). During this experiment, you will investigate π by measuring several aluminum disks and experimentally verifying the value of π. You will also determine the volume of a cylinder and use your volume and a measurement of the mass of the cylinder to determine the density of the cylinder.

During this lab, you will be exposed to several measuring devices. In addition to a meter stick, you will have the opportunity to use a micrometer, a vernier caliper, and a triple beam balance (scale). Your lab instructor will go over each instrument in detail; you should, however, keep several ideas in mind. While you perform the experiments, try to compare the different measuring devices and make a mental list of which instruments are better suited to the different tasks that you perform. Good measurements are at the heart of experimental physics, and now is as good a time as any to begin developing the physical intuition that will help you make good measurements.

Procedure:

Part I. Exercises on Significant Figures and Uncertainties

A. Practice Problems

At the end of the Introduction, you will find two sheets with exercises designed to help you work with significant figures and uncertainties. You will complete Part I (the practice problems) in class; make sure that the lab instructor checks them before you begin the experiment.

B. Turn-In Problems

The page after Part I (Part II) contains exercises that are similar to the ones that you completed in lab. You should do these problems on your own and turn them in with your lab report.

Part II. Measurement and Uncertainty

A. Measuring a disk

Your lab setup should include six aluminum disks. For each of the disks, measure the circumference (c) and the diameter (d). Use the meter stick and be sure to record your results to the nearest 0.1 mm. (Hint: there is an easy way to do this, be sure to use the supplies that you are given!). Record your results on the data sheet in table I-A.

Next, choose one of the aluminum disks and measure c and d six times each (on the same disk). Again, use the meter stick, and record your results to the nearest 0.1 mm. This time, record your data in table I-B.

B. Measuring a cylinder

Using the small chrome-plated brass cylinder from the measurement kit, as well as the vernier caliper, measure the diameter (d) and the length (ℓ) of the cylinder six times. Record your data in table II-A. Don't forget to record the least measure of the caliper.

Repeat this experiment using the micrometer instead of the vernier caliper. Again, be sure to use the chrome-plated cylinder and measure d and ℓ six times each. Record your results in table II-B.

C. Mass measurement

Using a triple beam balance, measure the mass of the brass cylinder three times. Record these measurements in table III.

Data Analysis:

Part I. Determination of π

A. Using the data from table I-A, plot a graph of the circumferences of the disks versus the diameters of the disks. Next, draw a best-fit slope onto the graph (that is, estimate the line that is closest to all of the data points). While it is not required for this lab, graphing software (such as Kaleidagraph) can automatically find the best-fit line. Next, draw two other lines that represent the maximum and minimum slopes that could fit the data reasonably well. Note that the slope of your original line is π and that the two other slopes (lets call them m_1 and m_2) allow us to calculate the error associated with the slope (and with π) with the relationship $\Delta\pi = \Delta m = .5*|m_2 - m_1|$. Calculate and record your value for $\pi \pm \Delta\pi$ below table I-A.

B. Next, using the data from table I-B, calculate c ± Δc and d ± Δd. Using these results, and remembering to propagate your error, calculate $\pi \pm \Delta\pi$ from the formula $\pi \pm \Delta\pi = \frac{c \pm \Delta c}{d \pm \Delta d}$. Record your results below table I-B.

C. Compare the results of your experiments in part A and B. Do the measurements agree with the accepted value of π? Do they agree with each other? Which method yields the more precise result.

Part II. Volume determination

A. Using the data that you gathered with the vernier caliper and recorded in table II-A, compute the mean and the standard deviation for d and ℓ. Using these results, and the formula for the volume of a cylinder ($V = \pi r^2 \ell$, where r is the radius of the cylinder), compute the volume of the brass cylinder. Record the results below table II-A.

B. Repeat the above calculation using the data from the micrometer. Record the results below table II-B.

C. Do the results in parts A and B agree with each other? Which method yields the more precise result?

Part III. Density determination

A. Using the three mass measurements of the brass cylinder (in table III), calculate and record M ± ΔM.

B. Using the results of part A, as well as the results from part B of the volume determination (the data associated with the micrometer), use the following definition of average density to compute $\rho \pm \Delta\rho$: $\rho = \frac{M}{V}$. Record the result next to your mass calculation.

C. Compare the density that you calculated with a typical value for brass of 8.5 g/cm^3.

Questions:

1) Is it possible for a series of measurements to give an accurate result that is imprecise? Explain.

2) Is it possible to have a precise result that is inaccurate? Explain.

3) How do the different measuring devices compare? Which device gave the smallest uncertainty in a length measurement? Which one was the least precise?

Data Sheet – Lab 5: Measurement, Error, and Uncertainty

Name (Observer) ____ Date _________

(Partner) ____ Section _______

I. Determination of π

A. Aluminum Disks

d (cm)	c (cm)

$\pi \pm \Delta\pi$ = _______________

B. Aluminum Disk

Trial	d (cm)	c (cm)

$d \pm \Delta d$ = __________________

$c \pm \Delta c$ = __________________

$\pi \pm \Delta\pi$ = __________________

Data Sheet – Lab 5: Measurement, Error, and Uncertainty

II. Volume of a brass cylinder

A. Vernier Caliper: least measure = __________

ℓ (cm)	d (cm)

$d \pm \Delta d$ = _______________

$\ell \pm \Delta\ell$ = _______________

$V \pm \Delta V$ = _______________

B. Micrometer: least measure = __________

ℓ (cm)	d (cm)

$d \pm \Delta d$ = _______________

$\ell \pm \Delta\ell$ = _______________

$V \pm \Delta V$ = _______________

III. Density of brass cylinder

Mass (g)

$M \pm \Delta M$ = _______________

Density = _______ ± ________

Experiment L-6: Gravity and Acceleration

Purpose: In this lab, you will analyze the motion of a freely falling body. During the course of your analysis, you will experimentally determine the acceleration of an object near the surface of the earth (g, that is).

Introduction: During this experiment, you will use the Behr Free Fall Apparatus. The Apparatus consists of a vertical column; an object falls along this column. As the object falls, a spark timer emits a spark at regular intervals; this spark passes through the object. Wax paper is situated in between the object and the cable that emits the spark, so the spark also passes through the paper. Notice that the object is accelerated (e.g., it is accelerating) and the time intervals between the sparks is constant – among other things, this tells us that the distance between the marks on the tape will get larger as our projectile falls further. Because we know the time interval, and because we can measure the distance between marks on the wax paper, we can use graphical or analytical methods to determine the acceleration and initial velocity of the projectile. Because we are close to the surface of the earth, the value that we obtain for the projectile's acceleration should be a good approximation of g.

Suppose that our object has an acceleration a and initial velocity v_0. Assuming that our projectile falls with uniform acceleration, we may use the following kinematic equations

$$v = v_0 + at \qquad (1)$$

$$s = v_0 t + \tfrac{1}{2}at^2 \qquad (2)$$

$$\frac{s}{t} = v_0 + \tfrac{1}{2}at \qquad (3)$$

Because we are considering free fall acceleration, a = g, where $g = 9.80\ \mathrm{m/s^2}$. During our experiment, we will verify these three equations and determine the numeric values of v_0 and g.

Procedure:

Part I – Making the tape record

A. Put the wax tape into position by running it along the column of the Behr Apparatus, through the opening at the top, and a few inches down the back of the column. Place the metal clip on the end of the tape to hold it in place. Next, close the switch on the electromagnet, and secure the projectile at the top of the column using the electromagnet. After securing the projectile, it should oscillate for a minute or two. After it has stopped moving, turn on the spark timer (your lab instructor will demonstrate proper operation of the spark timer). You should see sparks traveling between the ring on the projectile, through the wax paper, and to the column. Next, release the object by opening the switch on the electromagnet. The projectile will fall to the bottom of the column, leaving marks in the wax paper as it goes. After the projectile gets to the bottom of the column, turn off the spark timer and remove the wax paper from the Apparatus.

B. Examine the wax paper to make sure that the dots are clearly visible, and that you can distinguish them from any other marks on the paper. If the wax paper is not usable, follow the procedure again to generate a new piece of tape.

Part II – Preparing the tape for analysis

A. Place the tape on your lab table, white side up. Locate the first marks on the tape (they should be in a cluster, because the spark timer was on while the projectile was not moving). Beginning several dots down from the first mark, circle every other dot on the wax paper. Because the spark timer sparks 60 times every second, and we are looking at every other dot, the interval between dots will be 1/30th of a second.

B. Starting with 0, and working your way up (i.e., 0,1,2,…), label the dots that you circled. Note that we circle and label the dots in order to make sure that we can keep track of which marks are dots from the spark timer and which are stray marks.

Part III – Taking the data from the tape

A. Fasten the tape to the table with masking tape and place a double meter stick over the tape and next to the dots (so that the dots are next to the scale on the meter stick). Also, instead of starting at the beginning of the meter stick, place the 10 cm mark next to the dot you marked 0.

B. Record the (total) distance between dot 0 and each subsequent dot on the data table (in Part I, using the column labeled s). Make sure to subtract the 10 cm that we added in the last part. Also, notice that we have already labeled each dot with the appropriate interval (that is, o corresponds to the 0th interval, 1 corresponds to the 1st interval, etc.). Using this procedure, we are taking the time at the 0th dot to be $t = 0$.

Data Analysis:

Part I – Calculating the initial velocity and (average) acceleration

A. Using the data in Table I, plot a graph of s versus t (among other things, this is a graph of equation (2)). Your graph should look like a parabola. If it is a parabola, then we could use some advanced mathematical techniques or computer programs to obtain v_0 and $\bar{a}$, where $\bar{a}$ is the average acceleration (which should be a constant). Because our data contains some experimental error, we would have to use a regression technique to determine the best fit parabola; instead, we will take advantage of equations (1) and (3) to determine v_0 and $\bar{a}$.

B. When we examine equation (3), we notice that a plot of $\frac{s}{t}$ versus t should yield a strait line (assuming that a is constant). Graph $\frac{s}{t}$ versus t, and determine v_0 and $\bar{a}$ from the graph. v_0 should be the y-intercept, and (1/2) $\bar{a}$ should be the slope of the line (why?). Do not forget to calculate and propagate the error associated with your calculations, and be sure to show all of your work. Enter your results in Table II, and be sure to convert your units from intervals to seconds (where 1 int. = 1/30 sec.).

C. Next, we will consider the distance the projectile traveled in each interval (as opposed to the total distance traveled). Calculate this distance by subtracting each displacement from the succeeding one (that is, $\Delta s_{i-0.5} = s_i - s_{i-1}$). Record the values in the $\Delta s/\Delta t$ column between the lines used for t, s, and s/t. We record the results this way because we want to emphasize that Δs is the change in displacement during the entire interval, and not at t = 1,2, etc. Also note that because each $\Delta t = 1$ int., $\Delta s = \Delta s/\Delta t$, and $\Delta s/\Delta t$ is the average velocity (in cm/int.). Finally, for those of you who have had calculus, note that because our projectile is undergoing uniform acceleration, $\Delta s/\Delta t = ds/dt$ at the midpoint of the time interval. The upshot of this discussion is that we are able to treat Δs as a velocity corresponding to times in between the original intervals (that is, we are considering the velocities at t = 1/2, 3/2, … int.), and we are justified placing our results in a $\Delta s/\Delta t$ column corresponding to these times. In order to use this data to find v_0 and $\bar{a}$, plot a graph of $v = \Delta s/\Delta t$ versus time. If your graph is linear, then the acceleration of the projectile should be constant (why?). With the help of equation (1), determine v_0 and $\bar{a}$, where v_0 is the y-intercept of the graph, and $\bar{a}$ is the slope. Again, make sure to include error with your answers and show all of your work. Record your results in Table II.

Part II – Determining theoretical velocities

A. Use equation (1) and the results of your analysis from Part I (B) (your values for v_0 and $\bar{a}$) to calculate the velocity of the projectile at t = 1.5 intervals and t = 5.5 intervals. Record these values in Table III, and calculate the percentage difference between these values and your experimental values (found in Table I, last column).

B. Use equation (2) and the results of your analysis from Part I (C) to calculate the displacement (s) of the projectile at t = 2 intervals and t = 6 intervals. Record your results in Table III, and calculate the percentage difference between these values and your experimental value (found in Table I, s column).

Questions:

1) Do your two values of v_0 from Table II agree with each other? Justify your answer.

2) Do your two values of a=g (from Table II) agree with each other? Do they agree with the accepted value of $g = 9.8 \text{ m/s}^2$? Justify your answer.

Data Sheet – Lab 6: Acceleration and Gravity

Name (Observer) ____ Date _________

(Partner) ____ Section _______

Table I

t (int)	s (cm)	s/t (cm/int)	$\Delta s/\Delta t$ (cm/int)
0.0	0.0	▨	▨
0.5	▨	▨	
1.0			▨
1.5	▨	▨	
2.0			▨
2.5	▨	▨	
3.0			▨
3.5	▨	▨	
4.0			▨
4.5	▨	▨	
5.0			▨
5.5	▨	▨	
6.0			▨
6.5	▨	▨	
7.0			▨
7.5	▨	▨	
8.0			▨

Table II

Method	s/t vs. t	$\Delta s/\Delta t$ vs. t	Accepted Value
v_0 (cm/int)	±	±	▨
v_0 (cm/sec)	±	±	▨
a (cm/int^2)	±	±	▨
a (cm/sec^2)	±	±	980

Table III

	At t = 1.5 intervals	At t = 5.5 intervals
v(calc.) (cm/int)		
v(exptl.) (cm/int)		
Percentage difference		

	At t = 2 intervals	At t = 6 intervals
s(calc.) (cm)		
s(exptl.) (cm)		
Percentage difference		

Experiment L-7: Composition and Resolution of Forces (Force Tables)

Purpose: In this lab, you will learn to work with forces and associate forces as vector quantities. You will also practice solving vector equations using three different methods.

Introduction: In this experiment, we will be manipulating several masses (metal weights). We will investigate the tension caused when we suspend our masses from a ring (that is, we will hang all of the masses from one ring). The earth's gravitational field exerts a force on each mass; each mass applies a force on the string that holds them to the ring. We will also concern ourselves with the direction of the strings relative to the ring. The direction of each string, taken together with the tension in the string represents a vector (the magnitude of the vector is the tension) of force.

We will focus our attention on vectors constructed from weights and directions. After we construct three vectors, we will experimentally (and mathematically) determine a fourth vector called a resultant. Note that we define the resultant to be the sum of two or more vectors; in our case, because we have three vectors, the resultant is given by:

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} \qquad (1)$$

Our goal for the lab is to find $\vec{R}$ using three different methods. Bear in mind that the methods we will use here (excluding the experimental method) are general methods for solving almost any problems that contain vectors.

Procedure:

Part I – Experimental Method

In this part of the lab, we will use a bit of indirection to find $\vec{R}$. Experimentally, it is easy set up three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$. Unfortunately, $\vec{R}$ is not readily apparent from the three vectors; hence, we need a way to deduce it. We will determine $\vec{R}$ by finding a force that exactly counteracts (counterbalances) our three known vectors. This force is called the equilibrant $\vec{E}$, and it is related to $\vec{R}$ by the equation:

$$\vec{R} + \vec{E} = 0 \qquad (2)$$

Notice that (2) implies that $\vec{R}$ and $\vec{E}$ have the same magnitude but opposite directions.

A. Make sure that you write down the set of vectors that have been assigned to you by your T. A. (from table I) and level the force table with the water level.

B. Using the angles specific to your set of vectors, mount three pulleys on the perimeter of the table. Place the fourth pulley where you expect the equilibrium force will be.

C. Insert the pin at the center of the table and slip the ring over the pin. Take the strings coming off of the ring and thread them over the pulleys; suspend a hanger from each string. Add masses to hangers A, B, and C, (that is, the hangers that correspond to your known vectors) until each hanger has the appropriate amount of mass on it (do not forget the mass of the hanger).

D. Now change the direction of and add mass to the equilibrium hanger until the ring is centered about the pin. When the ring is properly centered, an imaginary ray extending from the end of any of the strings will pass through the top of the pin (see figure 1). To make sure that the ring is in equilibrium, displace the ring and make sure that it returns to the equilibrium position.

Figure 1: Position of the ring and pin at equilibrium.

E. Now we will determine the uncertainty $\Delta\vec{E}$ in the equilibrant. Place some additional mass on the equilibrium hanger (try 2 grams) and displace the ring from the equilibrium position. If the ring returns to the equilibrium position, repeat the procedure. Continue until the ring no longer returns to the equilibrium position. The mass that you have added is one half of the uncertainty in the magnitude of $\vec{E}$.

F. Next, remove the additional mass and make sure that the ring is in equilibrium. This time, vary the angle of the equilibrium hanger in increments of one half of a degree and displace the ring until it no longer returns to the equilibrium. Again, the total angular displacement is one half of the uncertainty in the direction of $\vec{E}$.

G. Use $\vec{E} \pm \Delta\vec{E}$ and equation (2) to find $\vec{R} \pm \Delta\vec{R}$. Make sure to record the results in your data table.

Part II – Mathematical Method

A. Calculate the x and y components of each of the three forces in your set of vectors, and record them in the appropriate table on your data sheet. Make sure to include the appropriate sign for each component.

B. Add the components together to obtain the net force in the x and y direction. Record the results in the data table.

C. Use the results of part B to calculate the resultant $\vec{R}$. Keep in mind that the inverse trigonometric functions can usually return two values, while your calculator will only provide one of these results. For example, $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$ or $\frac{3\pi}{4}$. Use your judgement to determine which value should be used in the problem. Record your results in the appropriate place on the data sheet.

Part III – Graphical Method

A. Use a ruler and a protractor to neatly draw a force diagram of your forces. Use the head to tail method to add your vectors (see figure 2) and make sure to draw the diagram to scale. Clearly label each vector and angle (starting each angle from the horizontal), and remember to record the scale that you use. The diagram should take up most of a page.

B. From your diagram, measure the length and angle of $\vec{R}$ and enter your results in the data sheet. Do not forget to estimate the uncertainty associated with your diagram.

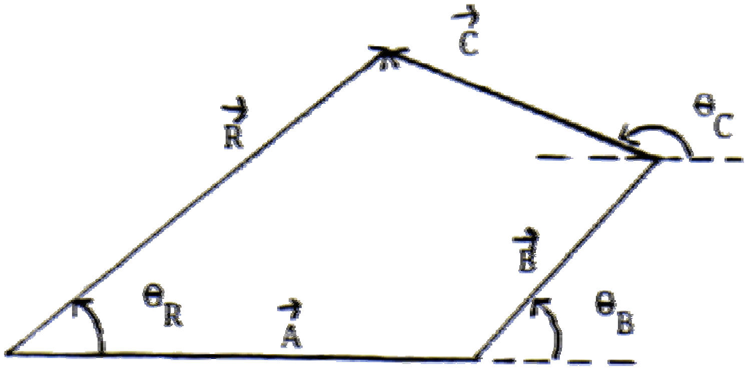

Figure 2: Head to tail method of adding vectors

Questions:

1) Describe the objective of the experiment and comment on what you accomplished.

2) Did the results from the three different sections agree with each other? Comment on any discrepancies.

Table I – Vector table

Problem Set	**Force A**		**Force B**		**Force C**	
	Mass (kg)	Angle (degrees)	Mass (kg)	Angle (degrees)	Mass (kg)	Angle (degrees)
1	0.100	0	0.135	120	0.240	250
2	0.200	0	0.150	120	0.100	210
3	0.120	0	0.085	150	0.150	240
4	0.150	0	0.120	130	0.200	320
5	0.220	0	0.100	110	0.160	260
6	0.300	0	0.225	140	0.375	210
7	0.120	0	0.140	150	0.150	250
8	0.180	0	0.140	120	0.100	320
9	0.100	0	0.080	130	0.120	300
10	0.150	0	0.100	120	0.200	300
11	0.200	0	0.120	110	0.150	250
12	0.320	0	0.220	120	0.260	210

Data Sheet – Lab 7: Composition and Resolution of Forces

Name (Observer) ____ Date _________

(Partner) ____ Section _______

	Problem No:			Mathematical	
	Mass	Force		Force Components	
		Magnitude (N)	Direction (degrees)	F_x (N)	F_y (N)
A					
B					
C					
				R_x=	R_y=

	Equilibrant $\vec{E}$			Resultant $\vec{R}$	
Method	Mass (kg)	Magnitude (Newtons)	Direction (Degrees)	Magnitude (Newtons)	Direction (Degrees)
Experimental				±	±
Mathematical	▨	▨	▨		
Graphical	▨	▨	▨	±	±

Experiment L-8: Velocity of a Projectile – The Ballistic Pendulum

Purpose: In this experiment, you will determine the velocity of a projectile using two separate methods: a ballistic pendulum and the trajectory of the projectile.

Introduction: A ballistic pendulum (see Figure 1) is a pendulum that shoots a projectile into an arm (called a pendulum bob) that is initially at rest. The projectile remains trapped inside of the arm, so the collision is an inelastic collision (think of shooting a bullet into a wooden block). Because the collision is inelastic, we know that kinetic energy is not conserved, and momentum is conserved. Note that as the pendulum bob moves, the kinetic energy from the projectile is transformed into gravitational potential energy.

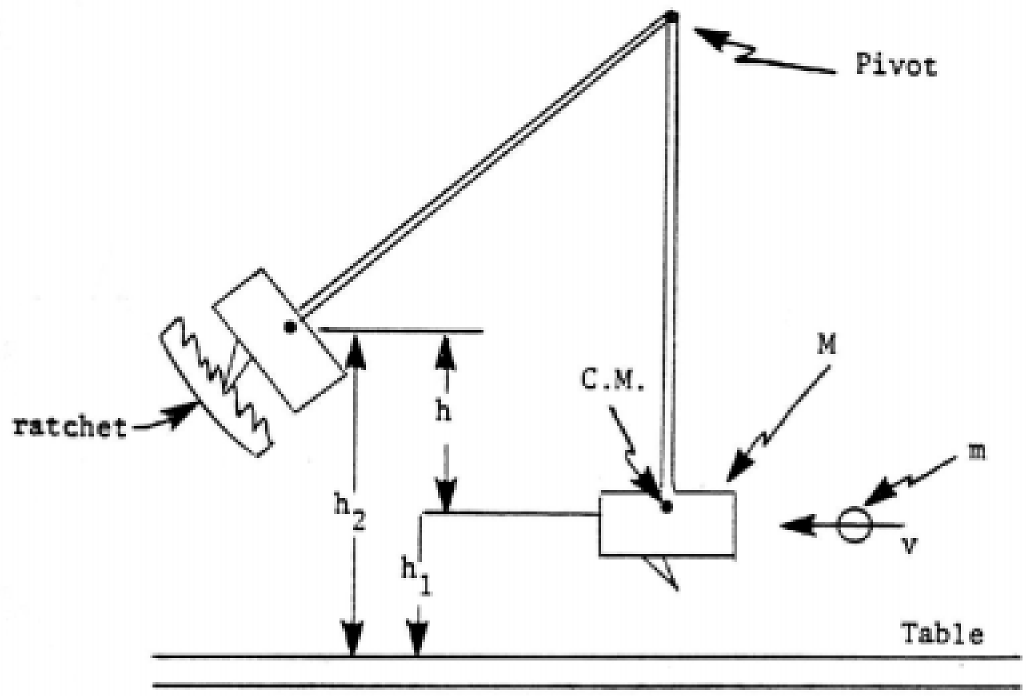

Figure 1: Essential features of the ballistic pendulum

As we see in Figure 1, the projectile, which has an initial velocity v_i and a mass m, runs into the pendulum bob with mass M. An inelastic collision occurs, and the projectile is trapped in the bob. Immediately following the collision, the bob and projectile move to the left with velocity V. Using the principle of conservation of momentum, we can write the initial velocity of the projectile as

$$v_i = V\frac{m+M}{m} \qquad (1)$$

So in order to determine the initial velocity of the projectile (v_i) we need to find the velocity of the projectile bob system immediately after the collision. As we noted earlier, the kinetic energy of the projectile bob system is transformed into gravitational potential energy, so we may use conservation of energy *after* the collision to determine V. Again referring to Figure 1, note that the pendulum bob swings up to a height h, where the ratchet locks it into its highest position (h). Applying the principle of conservation of energy (after the collision) yields:

$$V = \sqrt{2gh} \tag{2}$$

Next, we substitute (2) into (1) to determine the initial velocity of the projectile:

$$v_i = \frac{m+M}{m}\sqrt{2gh} \tag{3}$$

Notice that we can measure all of the quantities on the right hand side of (3) (m, M, and h, where g is a constant), so we may use this equation to determine the initial velocity of the projectile.

Now that we know how to use the ballistic pendulum to determine the initial velocity of our projectile, we would like to be able to check our results by determining the velocity in another manner. Luckily, we may independently measure v_i by shooting the projectile horizontally (and not into the pendulum, see Figure 2) and measuring the range and distance fallen by the projectile. We will call our independent measurement v_i'

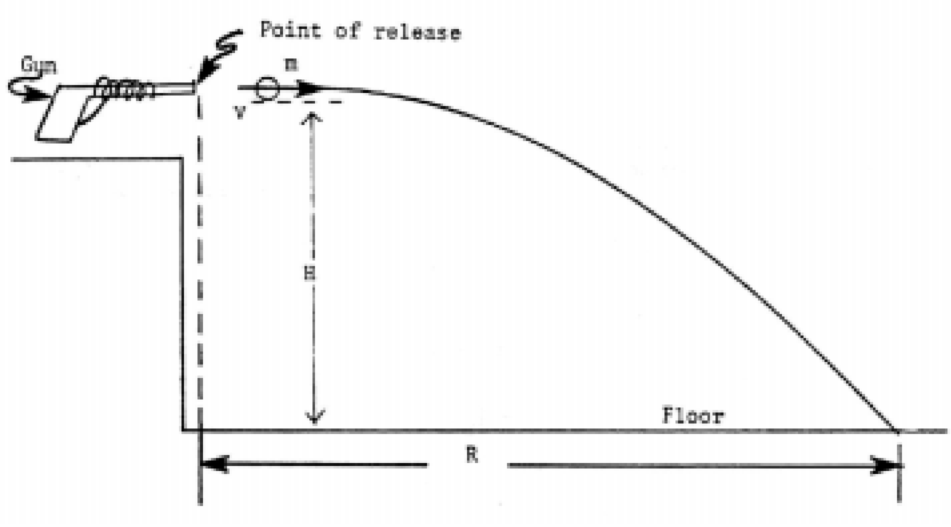

Figure 2: Setup for second measurement of projectile's velocity

If we shoot the projectile horizontally, then we may write its range as

$$R = v_i' t \tag{4}$$

where t is the projectile's time of flight. But t is also equal to the time it takes the projectile to fall a distance H (to the ground). So we may express the time as

$$t = \sqrt{\frac{2H}{g}} \tag{5}$$

Combining equations (4) and (5) yields

$$v_i' = R\sqrt{\frac{g}{2H}} \tag{6}$$

Now we may make an independent determination of v_i (which we have been calling v_i') based on the height and range of the projectile.

Procedure:

Part I – Determining the projectile's velocity using a ballistic pendulum

A. Make sure that the pendulum apparatus is level. The gun is cocked by placing the projectile in the projecting pin and pushing the pin back until it locks into place. There are three separate positions that you may lock the projectile into. Make sure that you use the same position throughout the experiment.

B. Determine the center of mass of the pendulum with the projectile in it by balancing the pendulum on the edge of a ruler. Mark the center of mass with a pencil.

C. Measure $h_1 \pm \Delta h_1$ by standing a ruler on the table or the base of the pendulum and sighting across to the center of mass. Take Δh_1 to be your estimate of how well you can determine h_1 (that is, an educated guess).

D. Launch the pendulum six different times, making sure to use the same locking position each time. Measure h_2 each time, then calculate the average h_2 and Δh_2. Take Δh_2 to be the standard deviation of h_2. Next, calculate $h \pm \Delta h = (h_2 \pm \Delta h_2) - (h_1 \pm \Delta h_1)$.

E. Disconnect the pendulum at the pivot, weigh the pendulum (with the projectile in it), and record the masses on the data sheet.

Part II – Determining the projectile's velocity using its range and fall

A. With the pendulum removed, place the apparatus near an edge of the table, making sure that there is a clear stretch of floor in front of the table. Test fire the apparatus to make sure that the projectile has enough room to land on the floor. For the rest of the experiment, make sure to cock the gun to the same setting that you used in Part I.

B. Place a piece of tape at the front edge of the apparatus so that it may be returned to the same location if it is moved. Next, take several more test shots to determine the projectile's approximate landing position. Tape a piece of white paper to the floor in the impact area. Cover this sheet with a piece of carbon paper, and then tape another piece of white paper on top of the carbon paper (the second sheet of white paper prevents the carbon paper from being torn when the projectile lands on it).

C. Shoot the projectile six times (that is, make sure that the projectile hits the paper six times). Remove the top sheet of white paper and the carbon paper, but leave the bottom sheet of white paper taped to the floor. Locate the point on the floor beneath the projectile's point of release (you may want to use a plum bob for this). Measure the distance from this point to the six impact marks and record them on the data sheet (under R). Calculate and record $R \pm \Delta R$, where ΔR is the standard deviation of R from the six shots.

D. Next, measure the height ($H \pm \Delta H$) from the gun to the floor, where ΔH is your estimate of the uncertainty in H. ΔH should be larger than 0.1 mm but smaller than 1 cm.

Data Analysis:

A. Use equation (3) and your data from Part I to calculate $v_i \pm \Delta v_i$. Record your result on the data sheet. Watch your units when you write down m, g, and h.

B. Use equation (6) and your data from Part II to calculate $v_i' \pm \Delta v_i'$. Record your result on the data sheet. To determine $\Delta v_i'$, you should propagate your uncertainties in R and H.

C. Using your values for v_i and v_i' to calculate the kinetic energy before and after the collision. Calculate the ratio of final K.E. to initial K.E.. Compare this result to the following theoretical ratio for a perfectly inelastic collision:

$$\frac{\text{K.E.}\left(\text{after collision}\right)}{\text{K.E.}\left(\text{before collision}\right)} = \frac{m}{m+M} \tag{7}$$

Questions:

1) Do $v_i \pm \Delta v_i$ and $v_i' \pm \Delta v_i'$ overlap? If they do not, give some reasons (remember that human error is not an appropriate reason). Comment on the kinetic energy results.

2) Using your value of v_i' and your value of V from equation (2), calculate the momentum of the system before and after the collision. Does momentum appear to be conserved in the collision?

3) Calculate the time of flight (the time the projectile was in the air) of the projectile in the range measurement.

4) Show logically how to get equations (1), (2), and (3).

5) Show logically how to get equations (4), (5), and (6).

6) Show logically how to get equation (7).

Data Sheet – Lab 8: Velocity of a Projectile – The Ballistic Pendulum

Name (Observer) ______________________________ Date _________

(Partner) ______________________________ Section _______

Mass of projectile ____________________ ± __________ g

Mass of pendulum ____________________ ± __________ g

Part I

Obs	h_2 (cm)	h_1 (cm)	h (cm)
1		▨	▨
2		▨	▨
3		▨	▨
4		▨	▨
5		▨	▨
6		▨	▨
Ave.	±	±	±

Part II

Obs	R (cm)	H (cm)
1		
2		
3		
4		
5		
6		
Ave.	±	

	Velocity (m/sec)
Part I (v_i)	±
Part II (v_i')	±

Energy	Energy (Joules)
Before Collision	
After Collision	
Experimental Ratio	
Theoretical Ratio	

Physics for Science and Engineering I

(Physics 110) Lab. Manual

by

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