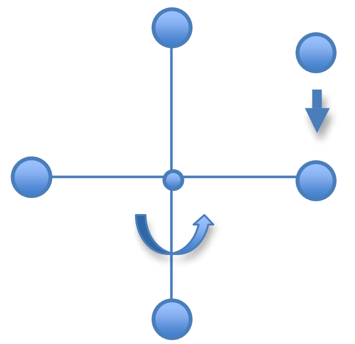
PHY 110 Exam 3 Solution

1. (8 pts) Given the vectors $\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{B} = -\hat{i} - 2\hat{j} + 3\hat{k}$, find the vector cross product $\vec{A} \times \vec{B}$.

a) $3\hat{i} - 4\hat{j} - 9\hat{k}$ b) $6\hat{i} - 12\hat{j} + 3\hat{k}$ c) $-2\hat{i} - 6\hat{j} + 2\hat{k}$ $\boxed{\text{d) } -\hat{i} - 10\hat{j} - 7\hat{k}}$



2. (8 pts) A 1 kg ball is dropped onto one of the balls in the diagram. The four 1 kg balls are connected by light rods, each of which extends 1 m from the pivot in the center. The 4-ball-and-stick system is rotating counter clockwise at 2 rad/s. The dropped ball reaches the ball on the right with a velocity of 5 m/s and sticks tightly to the ball with which it collides. What is the angular velocity of the ball-and-stick apparatus right after the collision?

a) 0.4 rad/s CCW b) 0.5 rad/s CW c) 0.6 rad/s CCW d) 0.8 rad/s CW

This is an "angular" collision, so total angular momentum is conserved. Take CCW to be the positive direction, recalling that $L = r_\perp mv$ *for the projectile,* $L = I\omega$ *for the rotating structure, and* $I = \sum_i r_i^2 m_i$ *for discrete masses. Thus the 4-ball structure has I = 4 kg•m*^{2} *and the 5-ball structure has I = 5 kg•m*^{2}*. Then we have*

$$L_i = L_f$$

$$(4)(2) - (1)(1)(5) = 5\omega_f \quad or \quad \omega_f = 0.6\frac{rad}{s}$$

3. (8 pts) The bicycle wheel is spinning counterclockwise when viewed from above (so as you view the picture, the right hand side of the wheel is moving away from you). The man and stool are initially at rest. If the man now rotates the wheel's axis 180˚ so that his right hand is above the wheel, the wheel/man/stool system will

a) rotate counterclockwise as viewed from above.
b) rotate clockwise as viewed from above.
c) not rotate at all.
d) tip over.

4. (8 pts) A 1000-kg weight hangs from a cylindrical aluminum bar (12 mm in diameter). By what fraction does the length of the bar change when the weight is attached to it? The elastic modulus (or Young's modulus) for aluminum is
N/m^2.

a) 0.03% b) 0.06% c) 0.12% d) 0.18%

The tension force applied to the bar is 9810 N. The cross-sectional area of the bar is $\pi(0.006m)^2 = 1.131\times10^{-4}\,m^2$. *The question asks for strain,* $\varepsilon = \dfrac{\Delta L}{L_0}$ *expressed as a percentage.*

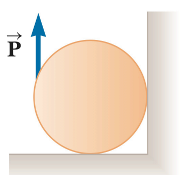
$$Y = \frac{\sigma}{\varepsilon} = \frac{F/A}{\Delta L/L_0} \text{ so that } \frac{\Delta L}{L_0} = \frac{F}{AY} = \frac{9810\,N}{\left(1.131\times10^{-4}\,m^2\right)\left(7\times10^{10}\,\dfrac{N}{m^2}\right)} = 1.24\times10^{-3} = 0.12\%$$

5. (8 pts) Consider the earth to be a sphere with radius = $6.37\times10^6\,m$ and a mass of $5.97\times10^{24}\,kg$. The moment of inertia of a sphere about an axis through its center of mass is $\frac{2}{5}MR^2$. What is the angular momentum of the earth due to its daily rotation about its axis?

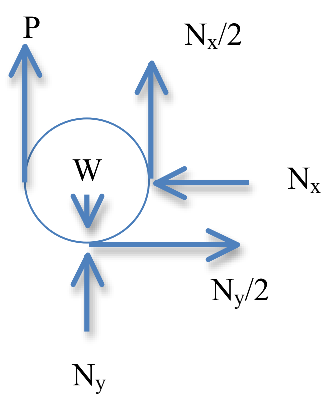
$\boxed{\text{a) } 7.0\times10^{33}\,\dfrac{kg\cdot m^2}{s}}$ b) $1.3\times10^{34}\,\dfrac{kg\cdot m^2}{s}$ c) $5.8\times10^{34}\,\dfrac{kg\cdot m^2}{s}$ d) $2.1\times10^{35}\,\dfrac{kg\cdot m^2}{s}$

Use $L = I\omega$ *with* $\omega = \dfrac{2\pi}{T}$ *where T is the number of seconds in one day.*

6. (14 pts) The uniform cylinder shown has weight W. If the coefficient of static friction between the cylinder and each surface is 0.5, what is the maximum value of the upward force P (in terms of W) that can be applied before the cylinder moves? *Hint: Convince yourself that the cylinder cannot slip at one surface without simultaneously slipping at the other surfaces as well.*



Note there are two normal forces, and the two friction forces are both maximum (f = μN) since slipping occurs at both surfaces. Take sum of torques about center and get



$$+CW\sum\tau_{center} = PR - \frac{N_y}{2}R - \frac{N_x}{2}R = 0 \;\; or \;\; P = \frac{N_x}{2} + \frac{N_y}{2}$$

$$+\rightarrow\sum F_x = \frac{N_y}{2} - N_x = 0 \;\; or \;\; N_x = \frac{N_y}{2}$$

$$+\uparrow\sum F_y = P + N_y + \frac{N_x}{2} - W = 0$$

Eliminate N_x and N_y from these three equations, and you get $P = \dfrac{3}{8}W$.

7. (8 pts) For an object undergoing simple harmonic motion,

a) velocity and acceleration can simultaneously be maximum, but must be of opposite signs.
b) velocity can never be negative when position is positive.
c) acceleration can never be positive when position is positive.
d) velocity and acceleration are always 180˚ (or π radians) out of phase.

8. (8 pts) A 0.8 kg mass on a spring undergoes simple harmonic motion with a period of 0.6 s. If the mass never gets more than 1 m from its equilibrium position, what is the maximum kinetic energy of the mass?

a) 16 J b) 23 J c) 44 J d) 57 J e) 69 J

Use m and k to get ω. *Then realize that the maximum kinetic energy is just the total energy of the system, which is* $\frac{1}{2}kA^2$.

9. (14 pts) A sinusoidal wave travels to the left in a horizontal string and has a wavelength of 0.4 m. A point on the string takes 0.05 s to make one complete oscillation of its vertical motion. A point on the string at x = 0 and t = 0 has a vertical position of 0.25 m above y = 0 and a downward velocity of 28 m/s. Write out the complete expression for the wave using the form $y(x,t) = A\sin(kx \pm \omega t + \phi)$. **Show all your work.**

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.4} = 5\pi \qquad \omega = \frac{2\pi}{T} = \frac{2\pi}{0.05} = 40\pi$$

$$y = A\sin[5\pi x + 40\pi t + \phi]$$

$$v_y = 40\pi A\cos[5\pi x + 40\pi t + \phi]$$

$$0.25 = A\sin\phi$$

$$-28 = 40\pi A\cos\phi \quad or \quad \tan\phi = -\frac{0.25}{28}40\pi \quad or \quad \phi = -0.843\,rad$$

This value for the phase constant is not consistent with $0.25 = A\sin\phi$, *so we must add* π *to get* $\phi = -0.843 + \pi = 2.30\,rad$. *Put this in* $0.25 = A\sin\phi$ *and get A = 0.335 m. The final expression is then*

$$y(x,t) = 0.335\sin[5\pi x + 40\pi t + 2.30]$$

10. (8 pts) A block attached to a horizontal spring executes simple harmonic motion. How many times during each complete oscillation is the kinetic energy of the block equal to the potential energy of the spring?

a) Never b) Once c) Twice $\boxed{\text{d) Four times}}$ e) Eight times

The block's kinetic energy is equal to the spring's potential energy somewhere between the equilibrium position and either maximum position (positive or negative). [Careful! This does not occur at $x = \pm\frac{A}{2}$*.] In one complete cycle (start at zero, go to positive maximum, back to zero, go to negative maximum, back to zero), the system passes such a point four times.*

11. (8 pts) A heavy rope hangs from the ceiling with the lower end free (not attached to any object). As a wave pulse travels down the rope from the attachment point at the ceiling toward the free end, the speed of the pulse

a) increases. $\boxed{\text{b) decreases.}}$ c) remains constant.

You should recognize this one as a quiz question, only backwards. As you go down the rope, the tension in the rope decreases, since it must support less mass hanging below that point. Since $v = \sqrt{\frac{T}{\mu}}$*, the speed also decreases as the pulse goes down the rope.*