



The wheel shown rotates so that the top of the wheel moves in the direction described as “into the page” and the bottom of the wheel moves “out of the page”. The wheel has large angular momentum and rotates freely on the thin shaft (the shaft itself does not rotate around its long axis). The right end of the shaft is suspended from the ceiling by a cable as shown.

1. The angular momentum vector of the wheel points in which direction?

- a) left      b) right      c) up      d) down      e) into page      f) out of page

*Use the right hand rule. Curl the fingers of your right hand in the direction of the rotation (so that they go over the top of the wheel, which is moving away from you), and your thumb points in the direction of the angular momentum vector.*

2. Consider the point where the cable attaches to the wheel’s shaft (right-hand end of the shaft). In what direction is the torque vector about that point produced by the weight of the wheel?

- a) left      b) right      c) up      d) down      e) into page      f) out of page

*The  $r$ -vector extends from the point where the cable attaches out to the center of mass of the wheel. Slide it along its own direction until the tail of the  $r$ -vector is at the center of mass, where the tail of the  $F$ -vector resides. The  $F$ -vector (the wheel’s weight) points down. Again using the right hand rule, curl the fingers of your right hand in the direction that moves the  $r$ -vector toward the direction of the  $F$ -vector, and your thumb points out of the page.*

3. If the wheel is released in the position shown, what will happen?

- a) The wheel and shaft will fall, rotating the shaft counter-clockwise through a large angle.  
 b) The wheel will remain fixed in the position shown.  
 c) The wheel’s shaft will remain horizontal, but the left end of the shaft will move into the page.  
 d) The wheel’s shaft will remain horizontal, but the left end of the shaft will move out of the page.

*Just like in the demonstration in class, the wheel’s angular momentum vector will remain mostly horizontal, but the end of the  $L$ -vector will move in the direction of the applied torque vector, which is the same as the direction of  $\frac{d\vec{L}}{dt}$ .*

4. Suppose I remove the cable and grab the wheel in the position shown, with my left hand on the left end of the shaft and my right hand on the right end of the shaft. Holding the wheel in the orientation shown, I then sit down on a stool that is free to rotate about a vertical axis. I then rotate the wheel's shaft so that it becomes vertical, with my left hand above my right hand. What happens?

a) Nothing.

b) Everything rotates around the vertical axis of the stool, clockwise when viewed from above.

c) Everything rotates around the vertical axis of the stool, counter-clockwise when viewed from above.

d) The entire universe will be destroyed in a cataclysmic matter-antimatter explosion.

*Again recall the demonstration in class. When I sit on the stool with the wheel's axis held horizontal, there is no vertical component of angular momentum in the system. But when I rotate the wheel so that my left hand is on top, the wheel's angular momentum vector now points upward, adding a vertical component to the system's angular momentum that wasn't there before. Since the total angular momentum vector of the system cannot change along the vertical axis (there are no external torques on the wheel/me/stool system on that axis), the system has to react by creating a downward angular momentum component. By the right hand rule, this requires everything to rotate clockwise when viewed from above.*

5. A stationary disk ( $I_{\text{disk}} = 0.1 \text{ kg}\cdot\text{m}^2$ ) is dropped onto a horizontal turntable ( $I_{\text{table}} = 0.4 \text{ kg}\cdot\text{m}^2$ ) that is freely rotating at 5 rad/s. When the disk stops slipping on the turntable, what will their angular velocity be?

a) 1 rad/s

b) 2 rad/s

c) 3 rad/s

d) 4 rad/s

e) 5 rad/s

*Since the only z-axis torque in the system is between the two disks (and therefore internal), the total angular momentum of the system in that direction is conserved. Consider just the z-component:*

$$L_i = L_f$$

$$I_{\text{turntable}}\omega_{\text{turntable},i} = (I_{\text{turntable}} + I_{\text{disk}})\omega_f$$

$$(0.4)(5) = (0.4 + 0.1)\omega_f$$

$$\omega_f = 4 \frac{\text{rad}}{\text{s}}$$