PHY 110 Spring 2018 Quiz 13 Solution

A transverse sinudoidal wave on a horizontal string travels to the **right** with a speed of 20 m/s. The wavelength of the wave is 1.5 m. A point on the string at x = 0 and t = 0 has a vertical position of 30 cm **below** $y = 0$ and a vertical velocity of 2.5 m/s **upward**. Assume the mathematical form of the wave is $y(x,t) = A\sin(kx \pm \omega t + \phi)$. Assume the usual positive directions for x and y.

1. What is the proper sign of the ωt term?

a) + **b) −** c) either is correct d) neither is correct

Use the minus sign for waves traveling to the right (toward positive x).

2. What is the value of k?

a) $\frac{\pi}{3}\ m^{-1}$ b) $\frac{2\pi}{3}\ m^{-1}$ c) $\pi\ m^{-1}$ **d)** $\frac{4\pi}{3}\ m^{-1}$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{1.5m} = \frac{2\pi}{\frac{3}{2}m} = \frac{4\pi}{3}\ m^{-1}$$

3. What is the value of ω?

a) $\frac{20\pi}{3}\ s^{-1}$ b) $\frac{50\pi}{3}\ s^{-1}$ **c)** $\frac{80\pi}{3}\ s^{-1}$ d) $\frac{100\pi}{3}\ s^{-1}$

$$v = \frac{\omega}{k} \quad so\ that \quad \omega = kv = \left(\frac{4\pi}{3}\right)(20) = \frac{80\pi}{3} s^{-1}$$

4. What is the value of A?

a) 0.128 *m* b) 0.224 *m* **c) 0.301 *m*** d) 0.493 *m*

5. What is the value of ϕ?

a) 1.51 rad b) 2.84 rad c) 3.77 rad **d) 4.61 rad**

Solve 4 and 5 in reverse order. Note that since $y = A\sin\left[kx - \omega t + \phi\right]$, *we can take a derivative with respect to time and get* $v_y = -\omega A\cos\left[kx - \omega t + \phi\right]$. *Now put in the information given in the statement of the problem to get*

$$-0.3 = A\sin\phi$$

$$2.5 = -\frac{80\pi}{3} A\cos\phi$$

Manipulate the two equations to get $\tan\phi = \frac{80\pi}{3}\frac{0.3}{2.5}$, *which gives* $\phi = 1.472\,rad$. *However, if you substitute this value into the first equation above, you can't get a positive value for A as required. Therefore you have to add* π *to get* $\phi = 4.61\,rad$. *Use this value in the first equation and find* $A = 0.301\ m$.

Possibly useful equations: $\omega = 2\pi f$, $f = \frac{1}{T}$, $k = \frac{2\pi}{\lambda}$, $v = \lambda f$, $v = \frac{\omega}{k}$