PHY 110 Spring 2018 Quiz 5 Solution

A 100 kg crate sits on the flat steel bed ($\mu_s = 0.75$) of a truck moving at a constant 20 m/s.

1. What maximum acceleration (or deceleration, in absolute value) can the truck undergo without making the crate slide?

a) $4.61\frac{m}{s^2}$ b) $5.28\frac{m}{s^2}$ c) $6.95\frac{m}{s^2}$ $\boxed{\textbf{d)}\ 7.36\frac{m}{s^2}}$

From a free-body diagram of the crate, the normal force N is just the weight of the crate, mg. Then the only horizontal force available to keep the crate from sliding is just the maximum friction force, $\mu_s N$*, which is just* $\mu_s mg$*. Put that lone force into Newton's second law (assume positive is in the direction of the acceleration), and you get*

$$\sum F = \mu_s mg = ma, \text{ or } \quad a = \mu_s g = (0.75)(9.81) = 7.36\frac{m}{s^2}$$

2. Assume the answer to #1 is $6\frac{m}{s^2}$ (it isn't, but pretend it is). What minimum stopping distance can the truck have without making the crate slip, assuming constant deceleration?

a) 23 m $\boxed{\textbf{b) 33 m}}$ c) 43 m d) 53 m

$$v^2 - v_0^2 = 2a\left(x - x_0\right) \quad 0^2 - 20^2 = 2\left(-6\right)\left(x - 0\right) \quad x = 33.3m$$

3. A coin spins on a flat turntable 0.10 m from the center at a constant 78 rpm (revolutions per minute). What minimum value of μ_s between the coin and the turntable is required to keep the coin in place as it spins?

a) 0.58 $\boxed{\textbf{b) 0.68}}$ c) 0.78 d) 0.88

The free body diagram is exactly the same as in problem 1, with the friction force pointing toward the center of the rotation, which is the direction of the radial (centripetal) acceleration. The right hand side of Newton's 2nd Law becomes $m\frac{v^2}{r}$*, so that* $\mu_s = \frac{v^2}{gr}$*. All you have to do is change the rotation speed to a linear speed:* $78\frac{rev}{\min} \times \left[\frac{2\pi\left(0.10\right)m}{1rev}\right] \times \left[\frac{1\min}{60s}\right] = 0.817\frac{m}{s}$*. Plug that value in for v and find that* $\mu_s = 0.680$.

4. A 40 g marble falls through a bottle of shampoo with a terminal speed of 0.5 m/s. If it is dropped from rest, how long will it take for the marble to reach a speed of 0.2 m/s?

a) 6 ms   b) 16 ms   **$\boxed{\text{c) 26 ms}}$**   d) 36 ms

From the formula given below, recall that $\frac{mg}{b}$ *is just* v_T*, the terminal velocity. Since the problem gives you* v_T *and m, you can calculate* $b = \frac{mg}{v_T} = \frac{(0.04)(9.81)}{0.5} = 0.7848$. *Now you can put that value in the formula for v(t), set v(t) = 0.2, and solve for t.*

$$v(t) = \frac{mg}{b}\left[1 - e^{-\frac{b}{m}t}\right]$$

$$0.2 = \frac{(0.04)(9.81)}{0.7848}\left[1 - e^{-\frac{0.7848}{0.04}t}\right]$$

$$0.2\frac{0.7848}{(0.04)(9.81)} = \left[1 - e^{-\frac{0.7848}{0.04}t}\right]$$

$$0.4 = \left[1 - e^{-\frac{0.7848}{0.04}t}\right]$$

$$e^{-\frac{0.7848}{0.04}t} = 0.6$$

$$-\frac{0.7848}{0.04}t = \ln(0.6)$$

$$t = 26.0 \times 10^{-3} s = 26ms$$

5. This is a "It's Friday – be happy!" question and costs you no points.

Possibly useful equations:

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 - v_0^2 = 2a(x - x_0)$$

$$f_s \le \mu_s N$$

$$a_{radial} = \frac{v^2}{r}\ inward$$

$$For\ \vec{F}_{drag} = -b\vec{v},\ v(t) = \frac{mg}{b}\left[1 - e^{-\frac{b}{m}t}\right]$$