PHY 110 Spring 2018 Quiz 9 Solution

1. The average car in the US is driven about 1,000 miles per month. If the radius of the typical tire is about 1 ft, through how many revolutions does a tire on a typical US car turn in one month? 1 mi = 5280 ft.

a) 230,000 $\boxed{\textbf{b) 840,000}}$ c) 5 million d) 22 million

Each time the wheel rotates once, the car moves a distance of one circumference of the wheel, $C = 2\pi r$. *In order for the car to move a distance D, the wheel must turn through N rotations, where* $N = \frac{D}{2\pi r} = \frac{(1000)(5280)}{2\pi(1)} = 840{,}338$

2. A grinding wheel slows down from an angular speed of 4000 rev/min to a stop in 600 revolutions. What was its angular acceleration, assumed constant?

a) –3.57 rad/s^2 b) –9.22 rad/s^2 c) –15.9 rad/s^2 $\boxed{\textbf{d) –23.3 rad/s}^2}$

First, do some unit conversions. $4000\frac{rev}{\min}\times\frac{2\pi\, rad}{1 rev}\times\frac{1\min}{60 s} = \frac{400\pi}{3}\frac{rad}{s}$ *and, by inspection, 600 rev = 1200* π *rad. Now use the stopping equation, assuming that the wheel is at* $\theta = 0$ *at the beginning of the problem and that* θ *is positive in the direction the wheel is spinning.*

$$\omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0)$$

$$0^2 - \left(\frac{400\pi}{3}\right)^2 = 2\alpha(1200\pi - 0)$$

$$\alpha = -\frac{160000\pi^2}{9}\frac{1}{2(1200\pi)} = -\frac{800\pi}{108} = -23.27\frac{rad}{s^2}$$

3. A disk (M = 10 kg, R = 0.6 m, I_{cm} = 1.8 kg m^2) rotates freely about a horizontal axis through its center. A 2 kg mass is tied to a rope, and the rope is wound around the circumference of the disk until the mass is suspended above the ground. If the mass is released, what is its downward velocity after it has fallen 1 m?

a) 1.96 m/s $\boxed{\textbf{b) 2.37 m/s}}$ c) 3.35 m/s d) 4.61 m/s

Since the disk never moves vertically, we don't have to worry about its gravitational potential energy. We place h = 0 for the hanging mass at its lowest position (at the end of the 1 m drop) and, since gravity is the only force doing work, we conserve total mechanical energy. It helps to realize that the vertical velocity of the hanging mass is the same as the tangential velocity of the edge of the disk, or $v = R\omega$.

$$E_i = E_f$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I_{disk,cm}\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}Mv^2 = \frac{v^2}{2}\left(m + \frac{M}{2}\right)$$

$$v = \sqrt{\frac{2mgh}{m + \frac{M}{2}}} = \sqrt{\frac{2(2)(9.81)(1)}{2 + \frac{10}{2}}} = 2.368\frac{m}{s}$$

4. The moment of inertia of a disk through its center is $\frac{1}{2}MR^2$. When the disk/mass system in problem 3 is in motion, what fraction of the system's total kinetic energy does the rotation of the disk represent?

a) 24% b) 46% **c) 71%** d) 88%

Note that E_f in the equations above is all kinetic energy. From the next to last line in the equations above, you can see that the kinetic energy can be written as $\frac{v^2}{2}\left(m + \frac{M}{2}\right) = \frac{v^2}{2}\left(2 + \frac{10}{2}\right) = \frac{v^2}{2}(2+5)$. *The first term represents the kinetic energy of the falling mass, and the second term represents the kinetic energy of the rotating disk. Thus the disk makes up 5/7 of the total kinetic energy, which is about 71%.*

5. What quantities are conserved in the motion described in problem 3?

a) The linear momentum of the hanging mass.
b) The kinetic energy of the disk.
c) The kinetic energy of the disk and hanging mass together.
d) The total mechanical energy of the disk/mass system.

The action of gravity makes the first three answers incorrect.

$$C = 2\pi r \quad s = r\theta \quad v_t = r\omega \quad a_t = r\alpha \quad I = \int r^2\,dm = \sum m_i r_i^2$$

$$\omega = \omega_0 + \alpha t \quad \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \quad \omega^2 - \omega_0^2 = 2\alpha(\theta - \theta_0)$$

$$K_{translation} = \frac{1}{2}mv^2 \quad K_{rotation} = \frac{1}{2}I\omega^2 \quad U_g = mgh$$