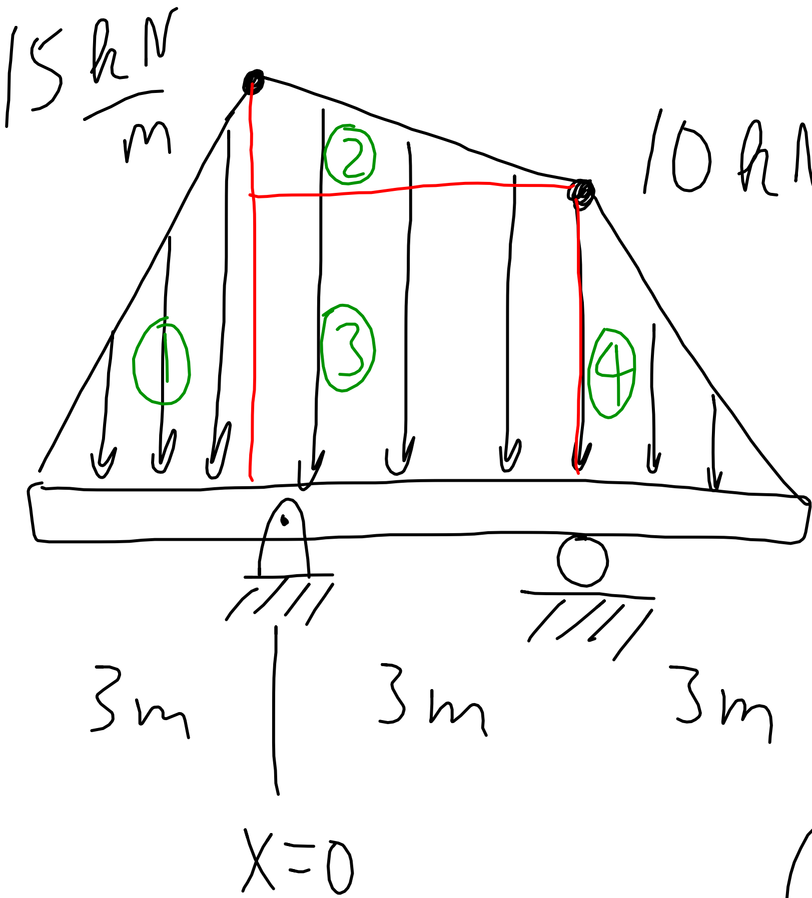


① 22.5 kN @ x = −1

② 7.5 kN @ x = 1

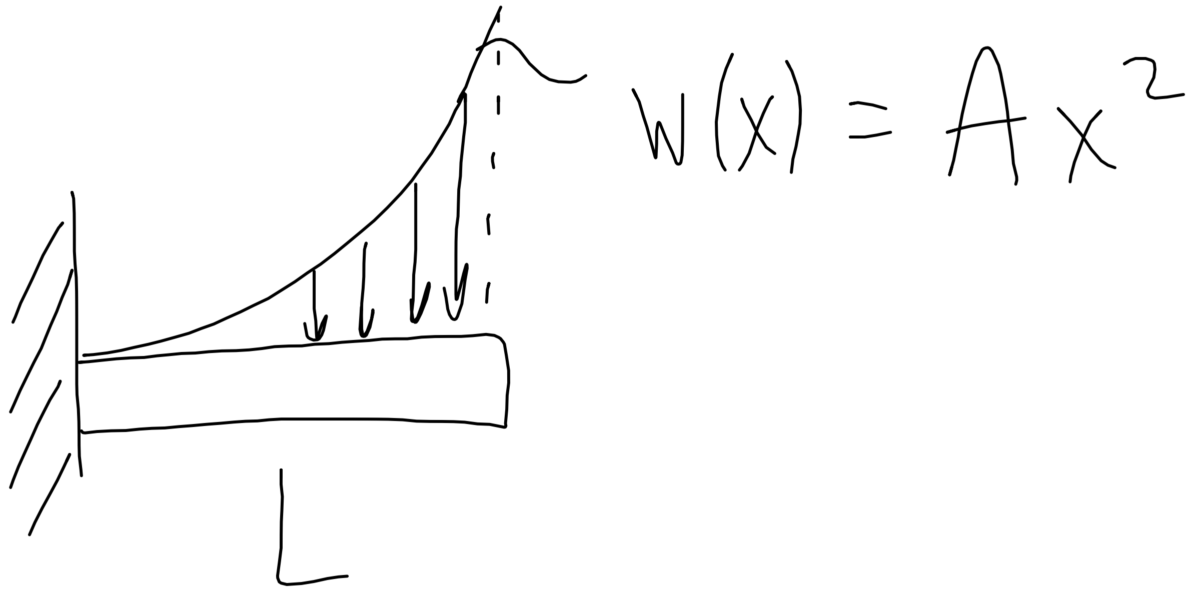
③ 30 kN @ x = 1.5

④ 15 kN @ x = 4

$F_R = 75\text{ kN}$

$\circlearrowleft \Sigma M_O = 22.5(-1) + 7.5(1) + 30(1.5) + 15(4) = 90\text{ kN}\cdot\text{m}$

$X_R = \frac{90}{75} = 1.2\text{ m}$

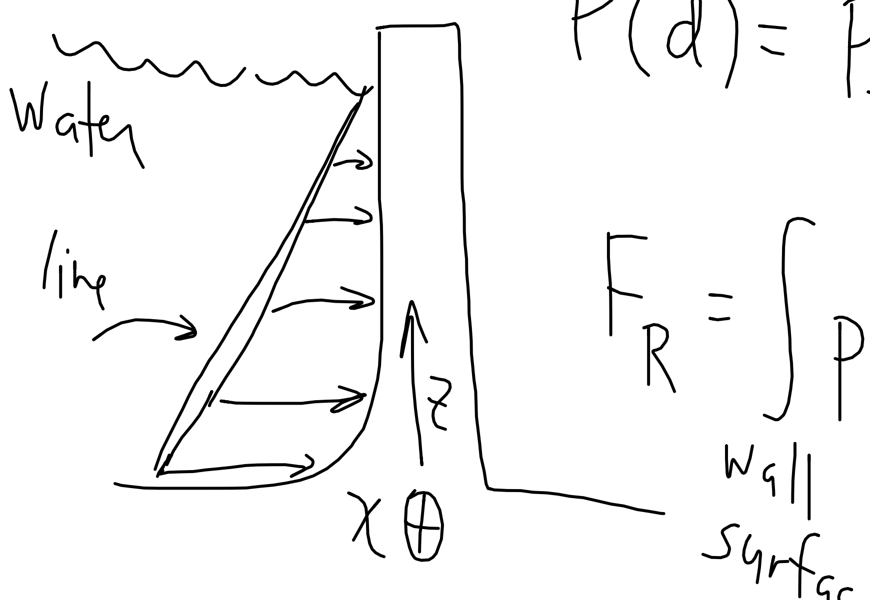


$$F_R = \int_0^L w(x)\,dx = \int_0^L Ax^2\,dx = \frac{Ax^3}{3}\Big|_0^L = \frac{AL^3}{3}$$

$$M_0 = \int_0^L x\,w(x)\,dx = \int Ax^3\,dx = \frac{Ax^4}{4}\Big|_0^L = \frac{AL^4}{4}$$

$$x_R = \frac{M_0}{F_R} = \frac{AL^4/4}{AL^3/3} = \frac{3}{4}L$$

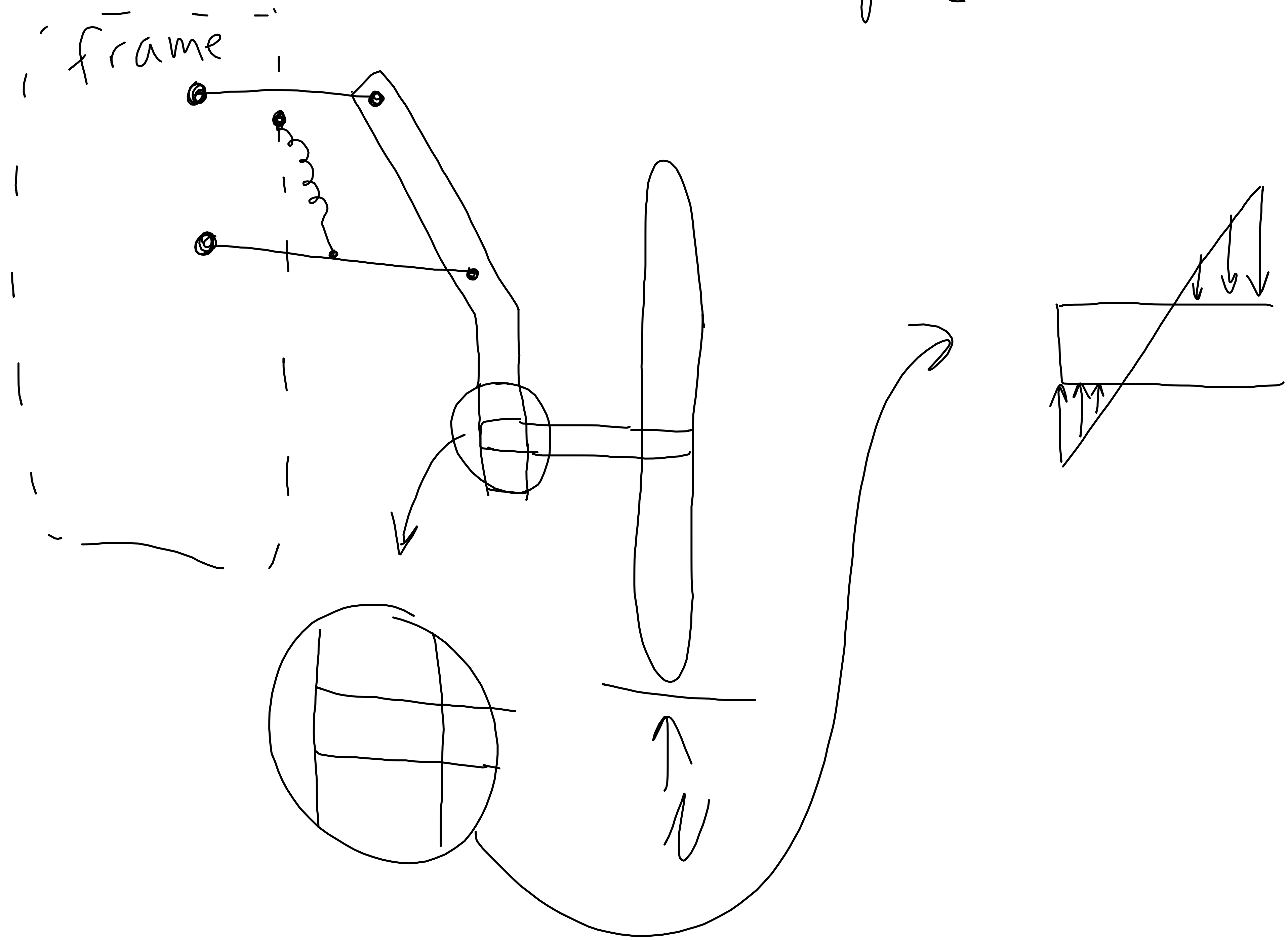
Practical distributed forces

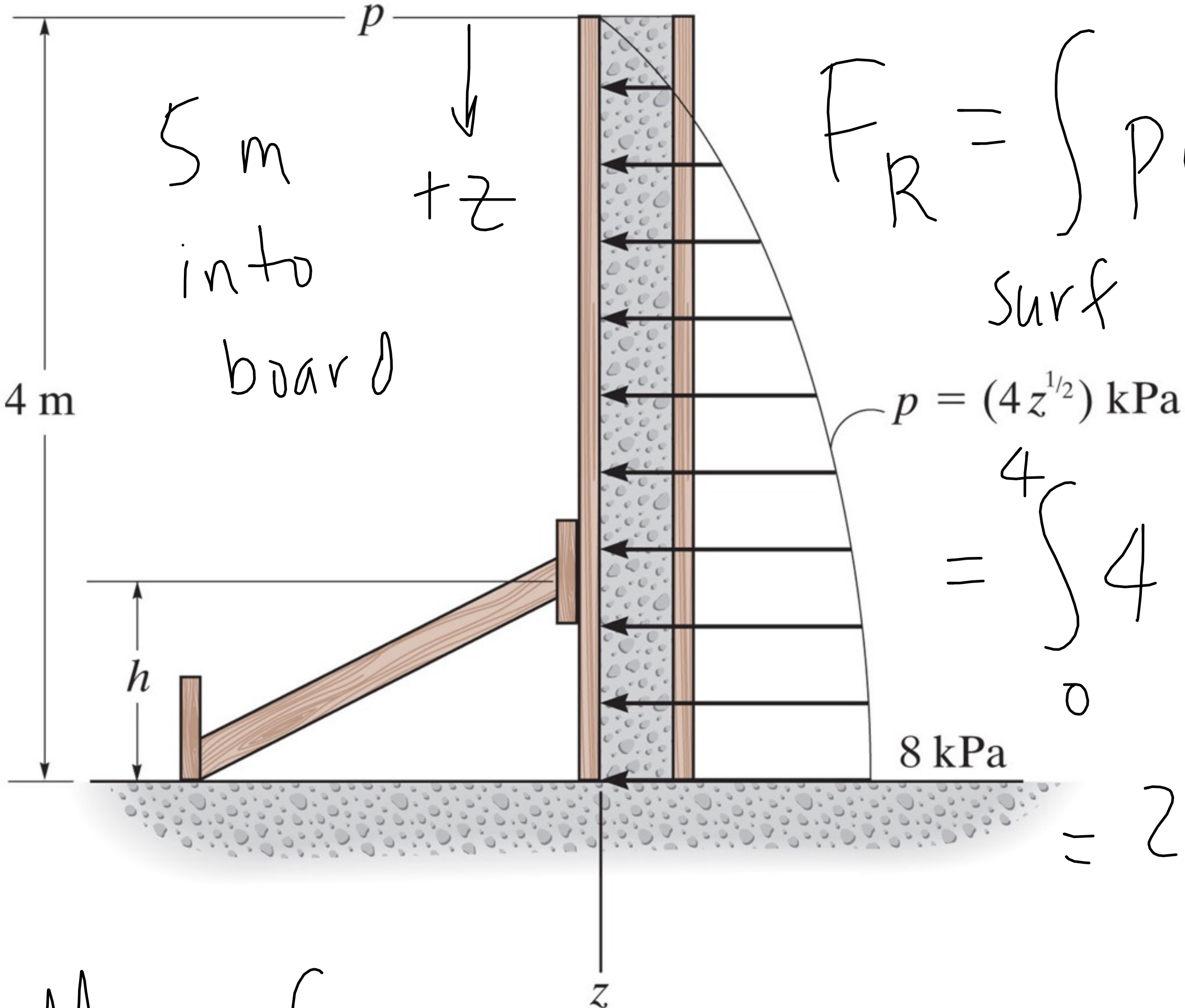


$$P(d) = P_{top} + \rho g d \qquad d = \text{depth}$$

$$F_R = \int p(x,z)\,dA \xRightarrow[\text{width}]{\text{uniform}} \int \underbrace{p(\overset{\text{out.}}{x},z)\,W}_{f(z)}\,dz$$

Solar car example





5 m into board

$\downarrow +z$

$$F_R = \int_{surf} p\,dA = \int \underbrace{p\,\overset{5\text{ m}}{W}}_{w(z)}\,dz$$

$$= \int_0^4 4z^{1/2}\,5\,dz$$

$$= 20\,\frac{z^{3/2}}{3/2}\Bigg|_0^4 = \frac{320}{3} = 106.\bar{6}\ \text{kN}$$

$$M_{top} = \int z\,5(4z^{1/2})\,dz = 20\,\frac{z^{5/2}}{5/2}\Bigg|_0^4 = 8\cdot 2^5 = 256\ \text{kN}\cdot\text{m}$$

$$z_R = \frac{256}{106.\bar{6}} = 2.4\ \text{m}$$ from top