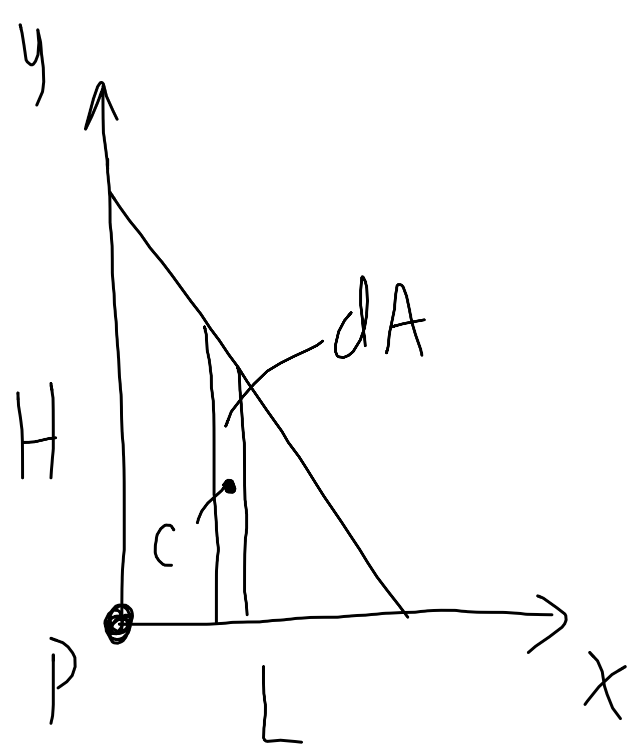
From last time: $I_{xy} = \int xy\, dA$



$$I_{xy,P} = I_{xy,C} + A\,\bar{x}\,\bar{y}$$

$$dI_{xy,P} = \cancel{dI_{xy,C}}^{\,0} + dA\,\underbrace{\bar{x}}_{x}\,\underbrace{\bar{y}}_{\frac{-\frac{H}{L}x+H}{2}}$$

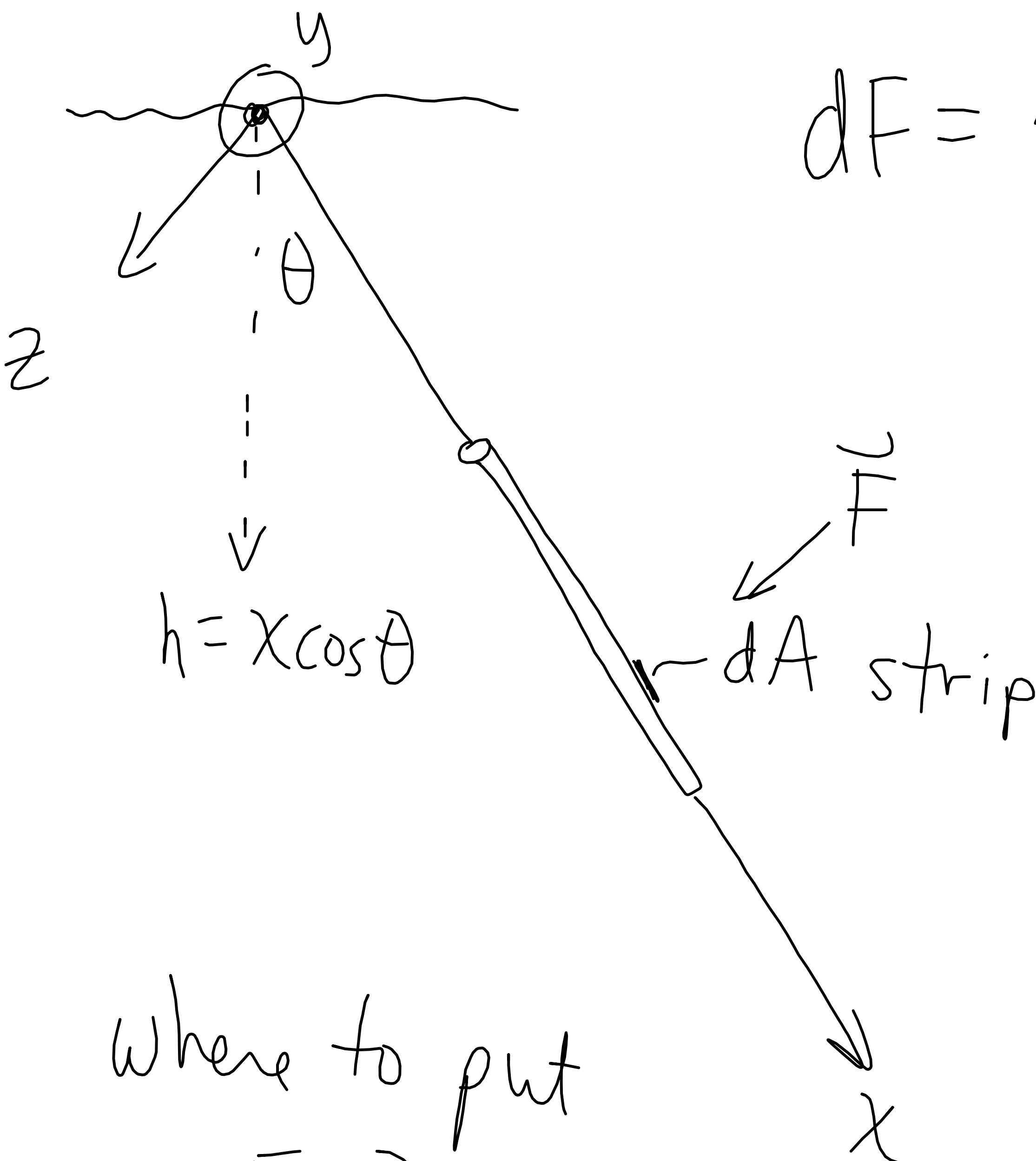
$$I_{xy,P} = \int x \,\frac{\left[-\frac{H}{L}x+H\right]}{2}\left[-\frac{H}{L}x+H\right] dx$$

$$I_{xy,p} = \int_0^L x \frac{\left[-\frac{H}{L}x + H\right]}{2} \left[-\frac{H}{L}x + H\right] dx$$

$$= \frac{1}{2}\int_0^L x\left(\frac{H^2}{L^2}x^2 - \frac{2H^2}{L}x + H^2\right) dx$$

$$= \frac{1}{2}\left[\frac{H^2}{L^2}\frac{x^4}{4} - \frac{2H^2}{L}\frac{x^3}{3} + H^2\frac{x^2}{2}\right]_0^L$$

$$= \frac{1}{2}\left[\frac{H^2L^2}{4} - \frac{2}{3}H^2L^2 + \frac{1}{2}H^2L^2\right] = \frac{H^2L^2}{24}$$



Where to put F_R?

$$dF = p\,dA = \rho g h\,dA$$

$$F_R = \int \rho g x \cos\theta \, dA$$

$$= \rho g \cos\theta \int x\,dA$$

$$= \rho g \cos\theta \; x_c A$$

$$= \rho g h_c A$$

$$= p_c A \;\checkmark$$

Find location $\vec{r}_{op}$ so that

$$\vec{r}_{op} \times \vec{F}_R = \int \vec{r} \times d\vec{F}$$

$$(x_p \hat{i} + y_p \hat{j}) \times p_c A \hat{k} = \int (x\hat{i} + y\hat{j}) \times p \, dA \hat{k}$$

$$\hat{i}: \quad y_p p_c A = \int y p \, dA$$

$$\hat{j}: \quad -x_p p_c A = -\int x p \, dA \longleftarrow$$

start here

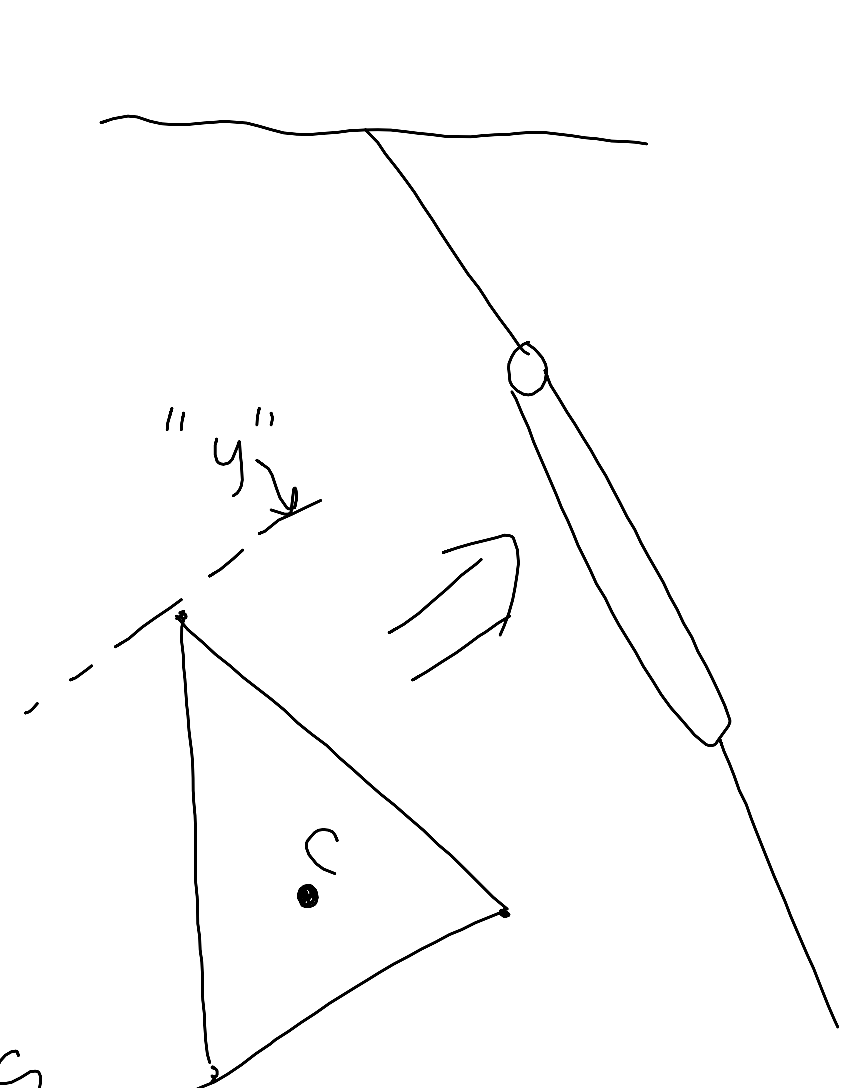
$$x_p = \frac{\int x p \, dA}{p_c A} = \frac{\int x \cancel{\rho g} x \cancel{\cos\theta} \, dA}{(\cancel{\rho g} x_c \cancel{\cos\theta}) \cancel{A}}$$

$$= \frac{\int x^2 \, dA}{x_c A}$$

$$= \frac{I_{y0}}{x_c A} = \frac{I_{yc} + A x_c^2}{x_c A}$$

$$x_p = x_c + \frac{I_{yc}}{x_c A}$$

$$x_p = x_c + \frac{I_{yc}}{x_c A}$$



x_p increases as I_{yc} increases

From $\hat{j}$ eqn

$$y_p P_c A = \int y P \, dA$$

$$y_p = \frac{\int y P \, dA}{P_c A} = \frac{\int y \cancel{\rho g} x \cancel{\cos\theta} \, dA}{(\cancel{\rho g} x_c \cancel{\cos\theta}) A}$$

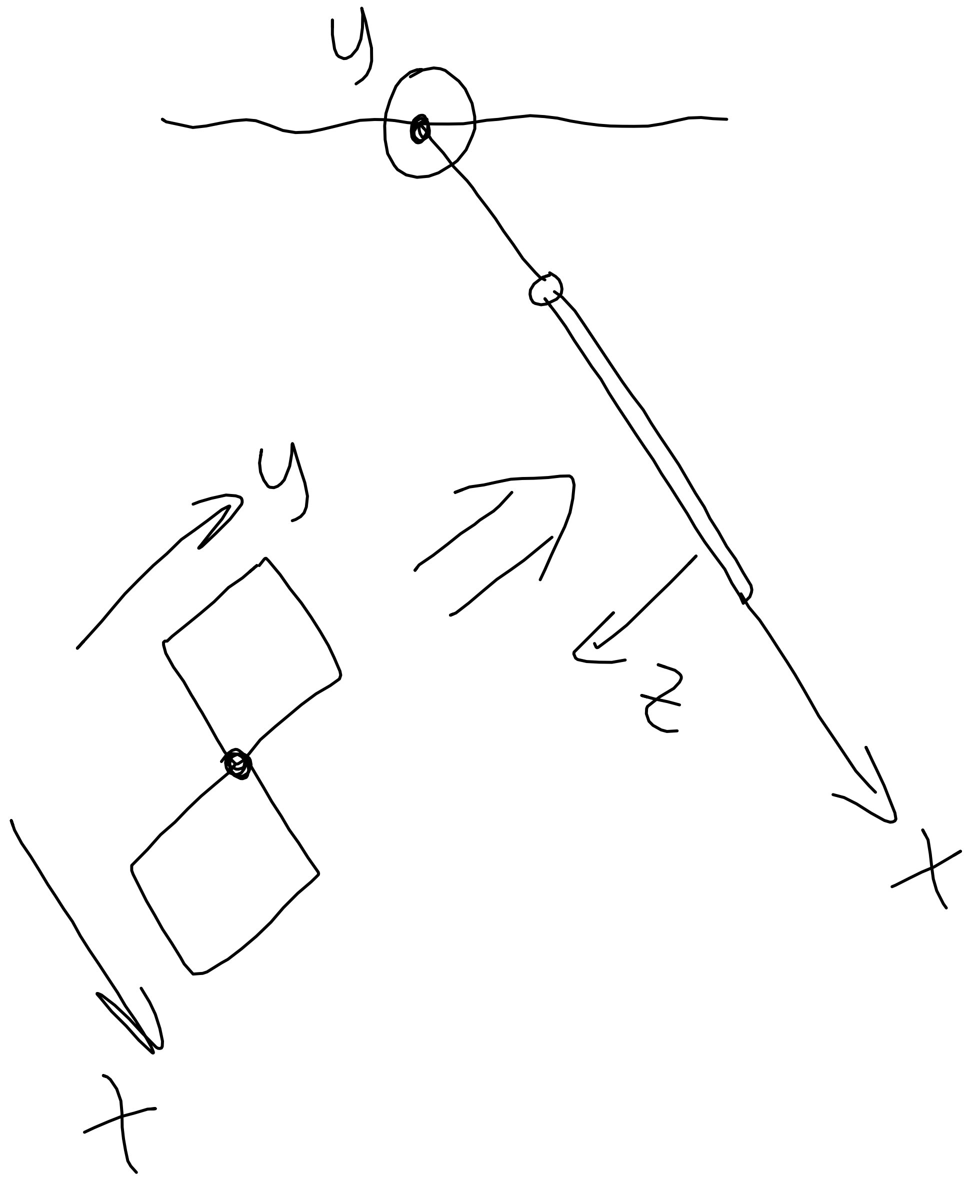
$$\boxed{y_p = y_c + \frac{I_{xy,c}}{x_c A}}$$

?

see next page

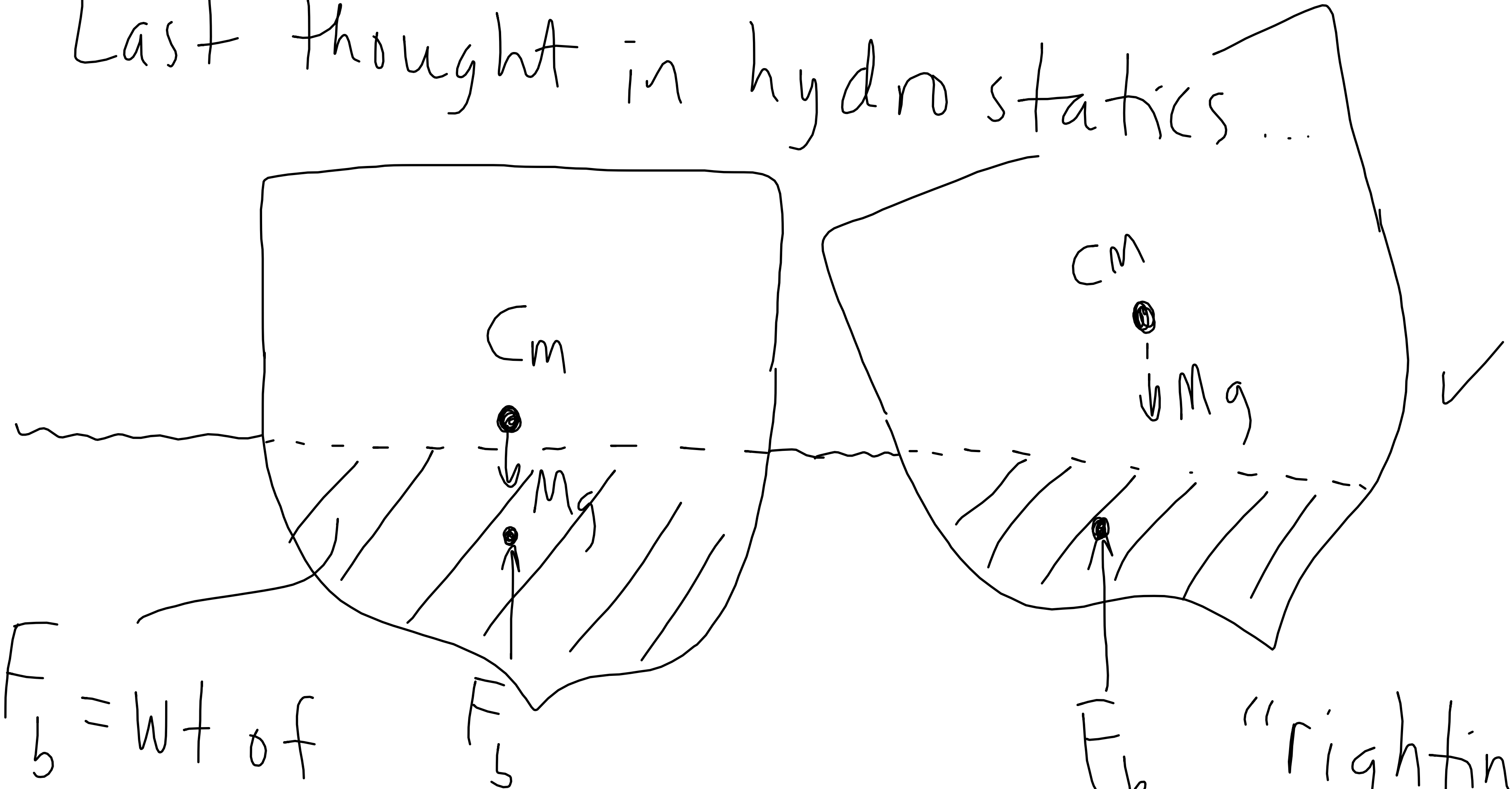
$$= \frac{\int xy \, dA}{x_c A} = \frac{I_{xy,0}}{x_c A}$$

$$= \frac{I_{xy,c}}{x_c A} + \frac{A x_c y_c}{x_c A}$$



y
y
z
x
x

Last thought in hydrostatics...



F_b = wt of displaced water @ centroid of displaced volume