Practice Exam 2 Solution

1. The tension T in the hoist line is 5 kN by inspection. The large pulley transfers both a horizontal and a vertical 5 kN force to the pin at B (left and downward). So a freebody diagram of boom AB has those two known forces at B, the tension in cable BC attached at B and acting up and to the left, and the two unknown pin reactions A_x and A_y at A.

Consider sum of moments at A:

$$+CCW \sum M_A = T_{BC}(\sin 17.745)(5) - (5)(5) = 0$$
$$T_{BC} = 16.405\, kN$$

$$+\uparrow \sum F_y = A_y + 16.405(\sin 17.745) - 5 = 0$$
$$A_y = 0$$

$$+\rightarrow \sum F_x = A_x - 16.405(\cos 17.745) - 5 = 0$$
$$A_x = 20.6\, kN$$

2. Note that there can only be a vertical (z) force at F, and that several reactions can exist at A: an x-force, a z-force, an x-moment, and a z-moment. Note that there can be no y-force or y-moment reaction at A.

$$\sum M_{y-axis} = 800(0.5) - F_y(0.8) = 0$$
$$F_y = 500\, N$$

$$\sum M_{x-axis} = M_{A,x} - 800(0.5) = 0$$
$$M_{A,x} = 400\, Nm$$

A quick calculation gives A_z = 300 N. By inspection, A_x and $M_{A,z}$ are both 0.

3. First, determine the external reactions.

$$+CCW \sum M_A = E_y(12) - 6(3) - 7(6) - 4(9) = 0$$
$$E_y = 8\, kN$$

From sum of forces in the y-direction, we get A_y = 9 kN. By inspection, A_x = 0.

Now make a vertical cut between H and G, since that cuts all three of the structural members whose forces you want to know. Let's consider the structure to the left of the cut and assume F_{BC} to the right (tension), F_{BG} up and to the right (tension) and F_{GH} down and to the left (compression).

$$+CCW\sum M_B = F_{GH}\left(\frac{2}{\sqrt{5}}\right)(3)-9(3)=0$$

$$F_{GH}=\frac{9\sqrt{5}}{2}=10.06\,kN\ (compression)$$

$$+\uparrow\sum F_y = 9-6-10.06\left(\frac{1}{\sqrt{5}}\right)+F_{BG}\left(\frac{3}{\sqrt{13}}\right)=0$$

$$F_{BG}=1.80\,kN\,(tension)$$

$$+\rightarrow\sum F_x = F_{BC}+1.80\left(\frac{2}{\sqrt{13}}\right)-10.06\left(\frac{2}{\sqrt{5}}\right)=0$$

$$F_{BC}=8.00\,kN\ (tension)$$

4. First, determine the external support reactions. The uniform part of the distributed force yields a 24 kN force half way between the pin supports, and the linear distribution gives a 12 kN force 4 m from the right end of the beam. Note that the boundary conditions for V are: -6 at left end, 0 at right end. For M, the boundary conditions are –20 at left end, 0 at right end. Call the left pin support A and the right pin support B.

$$+CCW\sum M_{left\ pin} = 20+6(6)-24(3)-12(8)+B_y(6)=0$$

$$B_y = 18.67\,kN$$

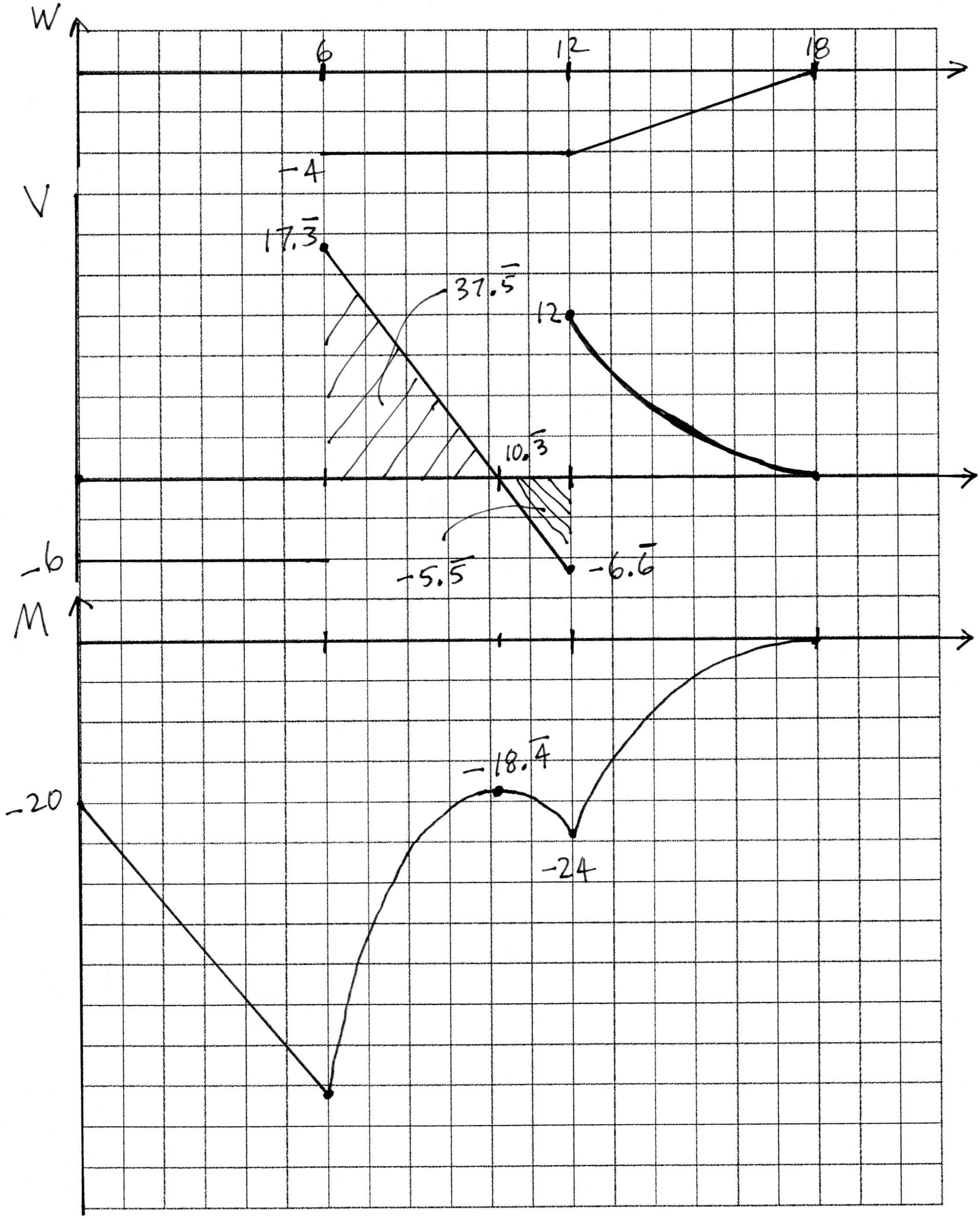
From sum of y-forces, A_y = 23.33 kN.

See the attached image containing w(x), V(x), and M(x) curves. Drawing the w(x) curve should be trivial.

For the V(x) curve, note that the 6 kN downward applied force at the left edge forms the boundary condition for that curve: V(x = 0) = –6. That continues until the jump caused by A_y, then the constant w value gives a constant slope (you should be able to calculate the endpoints, the place where the curve crosses the horizontal axis, and the areas of the two triangular regions). Then comes the jump due to B_y, then a parabolic region ending at zero (no shear at the right end of the beam). You should use the algebraic form of w(x) in that last region to calculate the exact form of V(x) in that region so that you can calculate the change in M over that region to verify that the M(x) curve really does end up at zero.

For the M(x) curve, M(x = 0) = –20, then a linear region down to –56, then a parabolic region. Note that the areas under the V(x) curve in this region give the change in M up to the local maximum and then to the next transition at x = 12 $(M=-18.\overline{4}\ at\ x=10.\overline{3}\ and\ M=-24\ at\ x=12)$. Then you get a cubic that goes from

$M = -24$ to $M = 0$, as required by the boundary condition at the right end of the beam. You should calculate the area under the $V(x)$ curve in this region to confirm that the change in M is indeed 24.


W
6
12
18
-4
V
17.$\bar{3}$
37.$\bar{5}$
12
10.$\bar{3}$
-6
-5.$\bar{5}$
-6.$\bar{6}$
M
-18.$\bar{4}$
-20
-24