Physics 217 Exam #1
Spring 2006

Name:______________________________________

1. Show that any function of the form $f(x+y+ct)$ is a solution of the equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial f^2}{\partial y^2} - \frac{2}{c^2}\frac{\partial^2 f}{\partial z^2} = 0$$

   (15 points)

2. Given that $\int_0^\infty \frac{dx}{(x^2+a^2)} = \frac{\pi}{2a}$ find $\int_{-\infty}^\infty \frac{dx}{(x^2+1)^2}$. Be carful of the limits. Give me an analytic solution. (No numerical results from calculators.) (10 points)

3. Consider the coordiante transformation given by

$$x = f(u,v,w) = uv\cos(w)$$
$$y = g(u,v,w) = uv\sin(w)$$
$$z = h(u,v,w) = \frac{1}{2}(u^2 - v^2)$$

   (a) Following the procedure developed in class, find the unit vectors $\hat{e}_u$,$\hat{e}_v$, and $\hat{e}_w$ Congratultions you have found the unit vectors for Paraboloidal Coordinates. (10 points)

   (b) Show that the unit vectors are all mutually perpendicular. (5 points)

4. Convert the differential operator $\frac{\partial f}{\partial \phi}$ from cylindrical to Cartesian coordinates (10 points)

5. Take the derivative with respect to x of

$$\int_{2x}^{x^2} \frac{x^2}{t^2} dt.$$

   Evaluate the result at the point $x = 2$.(10 points)

6. A quadrapole electric field is established by two positive charges of charge Q loacated at the points $(a,0,0)$ and $(a,\pi,0)$ and a two negative charges $-Q$ located at $(a,\frac{\pi}{2},0)$ and $(a,\frac{3\pi}{2},0)$ (All locations are given in cylindrical coordinates.)

   (a) Convert the locations of the charges into cartesian coordinates (x,y,z). (Hint: You can do these by inspection)(8 points)

   (b) Convert the locations of the charges in spherical coordinates (r,θ,ϕ). (Hint: You can do these by inspection)(8 points)

   (c) Determine the electrostatic potential at an arbitrary point (x,y,z) in space. (I would do this in Cartesioan coordinates if I were you.)(12 points)

   (d) Deterine the electic field fo the quadrapole for an arbitrary point along the z-axis, i.e. $(0,0,z)$. (I would again do this in Cartesioan coordinates.) (12 points)