Physics 217 Exam #3
Spring 2006

Name:____

1. The electrostatic potential generated by a particular charge distribution is found to be

$$\Phi(x, y, z) = \phi_0 e^{-\alpha x} \sin(\beta y)$$

   where ϕ_0, α, and β are constants

   (a) Find the electric field associated with this electrostatic potential. (10 points)

   (b) Calculate the charge density that generates the electric field.(10 points)

   (c) Is this electric field a conservative field? How do you know? (5 points)

   (d) Calculate the total electric charge found inside a cube of side $2a$ centered at the origin. (10 points)

2. A magnetic field near the axis of a fusion device called a Z-pinch may be written is cylindrical coordinates as

$$\mathbf{B} = B_0\left(\hat{\mathbf{e}}_z + \rho^2 \hat{\mathbf{e}}_\phi\right)$$

   where B_0 is the constant magnetic field strength on axis.

   (a) Use Ampere's Law ($\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$) to find the current density. (10 points)

   (b) Verify Stoke's Theorem for this B-field/current density by considering the integral form of Ampere's Law over a circular surface of radius a centered at the origin. ($\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \int\int \mathbf{J} \cdot d\mathbf{a} = \int\int \nabla \times \mathbf{B} \cdot d\mathbf{a}$)(10 points)

3. Consider the complex number $z = 2\sqrt{2} + 2\sqrt{2}i$.

   (a) Convert z into its polar form and sketch it location in the complex plane. (5 points)

   (b) Find the fourth roots of z and sketch their locations in the complex plane. (10 points)

4. Use general curvilinear coordinates to directly evaluate $\nabla \cdot (\nabla \times \mathbf{A})$, where $\mathbf{A}$ is a vector function. (10 points)

5. Consider the function $u(x, y) = x + y$

   (a) Show that $u(x, y)$ is a harmonic function (i.e. it satisfies the 2-D Laplace equation.) (5 points)

   (b) Find the function $v(x, y)$ such that $u(x, y) + iv(x, y)$ is an analytic function. (5 points)

6. Show that any function of the form $f(x/y)$ is a solution of the equation

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 0$$

   (10 points)

7. BONUS QUESTION: Let C be the boudary square whose sides lie along the lines $x = \pm 3$ and $y = \pm 3$. For the positive sense of integration evaluate the integral

$$\oint_C \frac{z^2}{(z-2)(z^2-10)} dz$$

   (5 points)