

Computer Assignment #2 2-D Projectile Motion with Drag

In the first computer assignment you learned the basic structure of the Runge-Kutta routine that we will use throughout the semester. With somewhat minor modifications, this code will allow you to solve a wide variety of differential equations. In this assignment, we will use it to solve for the range of a projectile thrown with a speed v at an angle θ with respect to the horizontal both with and without drag. Aside from the physics goal of learning more about projectile motion, a major computational goal is to get the student to start using additional loop structures for trying a lot of different initial conditions.

Case 1: No drag In this case the equations of motion are simply

$$\begin{aligned} \frac{dx}{dt} = v_x & \quad ; \quad \frac{dv_x}{dt} = 0 \\ \frac{dy}{dt} = v_y & \quad ; \quad \frac{dv_y}{dt} = -g \end{aligned}$$

If you assign your code variables as $Q(1) = x$, $Q(2) = v_x$, $Q(3) = y$, $Q(4) = v_y$ and $s = t$, your four equations become

$$\begin{aligned} \frac{dQ(1)}{ds} = Q(2) = fn(1) & \quad ; \quad \frac{dQ(2)}{ds} = 0 = fn(2) \\ \frac{dQ(3)}{dt} = Q(4) = fn(3) & \quad ; \quad \frac{dQ(4)}{dt} = -g = fn(4) \end{aligned}$$

Your assignment

1. Using an initial speed of 100 m/s, plot the trajectory (y vs. x) for the initial launch angles of 15° , 30° , 45° , 60° and 75° . (Put all of the curves on a single plot.)
2. Plot the range of the particle as a function of the launch angle from 1° to 89° , in 1° increments, again using an initial speed of 100 m/s. (I would advise using an additional loop to start change the angle.)

Case 2: Linear drag When we allow for linear drag, our equations of motion become

$$\frac{dx}{dt} = v_x \quad ; \quad \frac{dv_x}{dt} = -\kappa v_x$$

$$\frac{dy}{dt} = v_y \quad ; \quad \frac{dv_y}{dt} = -g - \kappa v_y$$

where $\kappa = c_1/m$. *Your assignment*

1. Using a value $\kappa = 0.01$, and again using an initial speed of 100 m/s, plot the trajectory (y vs. x) for the initial launch angles of 15° , 30° , 45° , 60° and 75° . (Put all of the curves on a single plot.)
2. Plot the range of the particle as a function of the launch angle from 1° to 89° , in 1° increments, again using an initial speed of 100 m/s and $\kappa = 0.01$.
3. Repeat part 2 using $\kappa = 0.001$ and $\kappa = 0.1$.

Questions

1. What angle gives the maximum range in the case of no drag?
2. What angle gives the maximum range when linear drag is included?
3. How well does the numerically calculated range agree with the weak drag approximation for the range that we did in class?

$$R = \frac{v_0^2}{g} \sin(2\alpha) - \frac{4\kappa v_0^3}{3g^2} \sin(\alpha) \sin(2\alpha)$$