

Computer Assignment #2 2-D Projectile Motion with Linear Drag

In the first computer assignment you learned the basic structure of the Runge-Kutta routine that we will use throughout the semester. With somewhat minor modifications, this code will allow you to solve a wide variety of differential equations. In this assignment, we will use it to solve for the range of a projectile thrown with a speed v at an angle θ with respect to the horizontal both with and without drag. Aside from the physics goal of learning more about projectile motion, a major computational goal is to get the student to start using additional loop structures for trying a lot of different initial conditions.

Case 1: No drag In this case the equations of motion are simply

$$\begin{aligned} \frac{dx}{dt} &= v_x & ; & & \frac{dv_x}{dt} &= 0 \\ \frac{dy}{dt} &= v_y & ; & & \frac{dv_y}{dt} &= -g \end{aligned}$$

If you assign your code variables as $Q(1) = x$, $Q(2) = v_x$, $Q(3) = y$, $Q(4) = v_y$ and $s = t$, your four equations become

$$\begin{aligned} \frac{dQ(1)}{ds} &= Q(2) = fn(1) & ; & & \frac{dQ(2)}{ds} &= 0 = fn(2) \\ \frac{dQ(3)}{dt} &= Q(4) = fn(3) & ; & & \frac{dQ(4)}{dt} &= -g = fn(4) \end{aligned}$$

Your assignment

1. Using an initial speed of 100 m/s, plot the trajectory (y vs. x) for the initial launch angles of 15° , 30° , 45° , 60° and 75° . (Put all of the curves on a single plot.)
2. Plot the range of the particle as a function of the launch angle from 1° to 89° , in 1° increments, again using an initial speed of 100 m/s. (I would advise using an additional loop to start change the angle.)

Case 2: Linear drag When we allow for linear drag, our equations of motion become

$$\frac{dx}{dt} = v_x \quad ; \quad \frac{dv_x}{dt} = -c_1 v_x$$

$$\frac{dy}{dt} = v_y \quad ; \quad \frac{dv_y}{dt} = -g - c_1 v_y$$

Your assignment

1. Using a drag coefficient of 0.01, and again using an initial speed of 100 m/s, plot the trajectory (y vs. x) for the initial launch angles of 15°, 30°, 45°, 60° and 75°. (Put all of the curves on a single plot.)
2. Plot the range of the particle as a function of the launch angle from 1° to 89°, in 1° increments, again using an initial speed of 100 m/s and a drag coefficient of $k = 0.01$.
3. As a homework problem you found that in the weak drag limit, $k = c_1/m \rightarrow 0$, the theoretical value for the range is give by

$$R_{theoretical} = \frac{2UV}{g} \left[1 - \frac{4\kappa V}{3g} \right]$$

where $U = v_0 \cos \theta$ and $V = v_0 \sin \theta$. (Note that as in the first computer assignment, it is an easy matter to calculate this function at the beginning of your program.) Plot a graph of the percentage difference between the calculated and approximate theoretical value as a function of the kT where T is the time of flight for the projectile.

4. Repeat part 2 and 3 using drag coefficients of 0.001 and 0.1.

Questions to consider.

1. What angle gives the maximum range in the case of no drag?
2. What angles gives the maximum range when linear drag is included?
3. Under what conditions does the theoretical calculation of the range agree with the calculated value?

Aside from the general write up for this lab, you should have 1) trajectory plots for the no drag case and the $k = 0.01$ case 2) a plot of the range as a function of the launch angle (I'd put all four cases on the same plot.) and 3) a plot of the percent difference between the theoretical and calculate ranges as a function of kT for the three values of k . (I'd put all three on one plot.)

If you are feeling particularly brave, an interesting extension of this lab would be to determine how the maximum range launch angle varies as a function of the drag coefficient.