Name:______________________________(1 point)

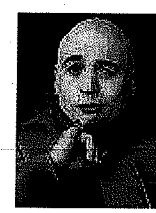
Physics 220
Exam 2, Fall 2011

1) (General 2-D Problem) New research discoveries on the planet Bovine have shown that the potential energy near the surface of the planet is given by

$$U(x,y)=\frac{1}{2}ky^2-mbx$$

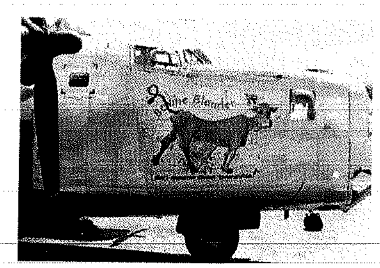
where m is the mass of an object in the potential, and k and b are both constants. (Assume that the x-direction is parallel to the ground and the y-direction is up.) The Holstein colony has developed a new weapon that can launch a manure bomb with an initial speed v_0 at an angle α with respect to the horizontal. (Keep in mind that the forces from the above potential are not going to give simple projectile motion like in a constant gravitational field but you should recognize the resulting equations.)

a) Using the potential given above, determine the force near the surface of planet Bovine.(6 points)
b) Is this force conservative? Explain. (3 points)
c) Calculate the x position of the manure bomb as a function of the time assuming that it starts from $x=0$, $y=0$ at $t=0$. (6 points)
d) Calculate the y position of the manure bomb as a function of the time assuming that it starts from $x=0$, $y=0$ at $t=0$. (6 points)
e) At what time does the manure bomb first strike the ground? (Do not count $t=0$ as the first time.) (6 points)
f) What is the range of the manure bomb? (6 points)



2) (Orbital Dynamics/Kepler's Laws/etc.) Dr. Evil is in a spacecraft of mass m that is in a circular orbit of radius R around the earth. In order to evade Austin Powers, he wishes to place it into a new orbit that has a radius 8 times that of the original orbit.

a) If the original orbit has a period τ, what will the period of the new orbit be (in terms of τ)? (7 points)
b) By how much would we have to increase the speed of the spacecraft in order to place it in a transfer ellipse that will take it out to the new orbit? Give your answer in terms of the original velocity in the low earth orbit. (You do not need to calculate the second change in speed once it reaches the new orbit.) (7 points)
c) What is the eccentricity of the elliptical transfer orbit (Hint: Consider fun with ellipses expressions for apogee and perigee)? (7 points)
d) For the elliptic transfer orbit, what is the velocity of the spacecraft at apogee (largest radius) in terms of the velocity at perigee (smallest radius)? (6 points)
e) What is the time required to go from the low orbit to the high orbit in terms of the period of the original low orbit? (i.e. what is the transfer time?) (6 points)



3) (Motion Relative to Earth/NIRF) During WWII, Raymond Zich's dad flew on a B24 bomber named the "Bovine Blunder" (true fact) in the South Pacific. During one particularly daring raid the bomber approached a target located on the equator (i.e $\lambda=0$) flying due North at a height of $h = 8000$ m and flying with an air speed of $v_0=125$ m/s. While approaching the target a cow is dropped out of the bomb bay door. (1 free point)
(Remember that $\omega = 7.3\times10^{-5}\ s^{-1}$.)

a) How long does it take the cow to hit the ground?(8 points)
b) How far does the cow travel in the North-South direction?(8 points)
c) How much does the cow drift in the East-West direction due to Coriolis acceleration?(8 points)
d) Is there any place on earth (i.e another λ) where the East-West drift would be zero for this initial height and velocity? If so, where? (8 points)

1) $U = \frac{1}{2} k y^2 - m b x$

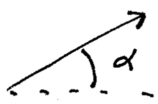
a) $\underline{F} = -\nabla U = -\frac{\partial U}{\partial x}\hat{e}_x - \frac{\partial U}{\partial y}\hat{e}_y - \cancel{\frac{\partial U}{\partial z}}\hat{e}_z$

$\cancel{\frac{\partial U}{\partial z}} \to 0$

$$\boxed{\underline{F} = m b\,\hat{e}_x - k y\,\hat{e}_y}$$

b) This is a conservative force. It is derived from a potential.

c) x-dir



$V_{ox} = V_o \cos\alpha$

$V_{oy} = V_o \sin\alpha$

$\cancel{m}\ddot{x} = \cancel{m} b$

$\ddot{x} = b \Rightarrow$ constant acceleration

$x = \cancel{x_o} + V_{xo} t + \frac{1}{2} b t^2$

$\cancel{x_o} \to 0$

$$\boxed{x = (V_o \cos\alpha) t + \frac{1}{2} b t^2}$$

d) $m\ddot{y} = -ky$

$\ddot{y} + \frac{k}{m} y = 0$ Define $\omega_o^2 = k/m$

$\ddot{y} + \omega_o^2 y = 0$ S.H.O.

$y = A \sin(\omega_o t)$

Find A $\dot{y} = \omega_o A \cos(\omega_o t)$; at $t = 0$ $\dot{y} = V_{oy}$

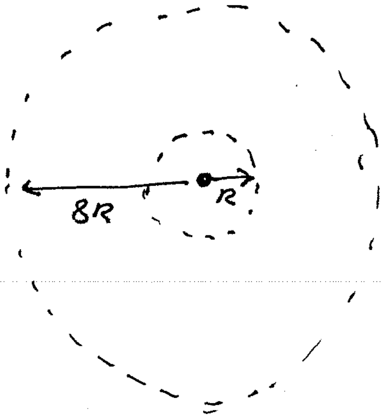
$V_o \sin\alpha = \omega_o A \Rightarrow A = \frac{V_o}{\omega} \sin\alpha$

$$\boxed{y = \frac{V_o}{\omega} \sin\alpha \sin(\omega_o t)}$$

e) Bomb Hits the ground $y = 0$; $\omega_o t = \pi \Rightarrow \boxed{t = \pi/\omega_o}$

f) $$\boxed{R = \frac{\pi V_o}{\omega_o}\cos\alpha + \frac{1}{2} b \frac{\pi^2}{\omega_o^2}}$$

2)



a) Kepler's Law

$$\tau^2 = C R^3$$

$$\tau_s^2 = C(8R)^3 = 8^3 C R^3$$

$$\tau_s^2 = 8^3 \tau^2$$

$$\boxed{\tau_s = 8^{3/2}\tau = 22.6\tau}$$

b) From class

$$\Delta V = V_{1t} - V_1 = V_1\left[\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right]$$

$$= V_1\left(\sqrt{\frac{16\cancel{R}}{9\cancel{R}}} - 1\right)$$

$$\Delta V = V_1\left(\frac{4}{3} - 1\right) = \boxed{\frac{V_1}{3} = \Delta V}$$

c)

$$r_p = R = \frac{\alpha}{1+\epsilon} \Rightarrow \alpha = (1+\epsilon)R$$

$$r_a = 8R = \frac{\alpha}{1-\epsilon} \Rightarrow \alpha = (1-\epsilon)8R$$

$$R + \epsilon R = 8R - 8\epsilon R$$

$$9\epsilon \cancel{R} = 7\cancel{R}$$

$$\boxed{\epsilon = 7/9}$$

d) At apogee/perigee $L = mrV$, since L is conserved

$$\cancel{m}\cancel{R}V_p = \cancel{m}(8\cancel{R})V_a$$

$$\boxed{V_a = \frac{1}{8}V_p}$$

e) Back to Kepler's Laws; Transfer time is ½ period of transfer ellipse. semi-major axis $a = \frac{1}{2}(9R) = 4.5R$

$$\tau_t^2 = C(4.5)^3 R^3 = (4.5)^3\tau^2$$

$$\tau_t = (4.5)^{3/2}\tau$$

$$\Rightarrow T = \frac{1}{2}\tau_t = \frac{1}{2}(4.5)^{1.5}\tau = \boxed{4.77\tau}$$

3) Motion Relative to Earth

$$x' = x_0' + \dot{x}_0' t - \omega t^2\left[\dot{z}_0' \cos\lambda - \dot{y}_0' \sin\lambda\right] + \frac{1}{3}\omega g t^3 \cos\lambda$$

$$y' = y_0' + \dot{y}_0' t - \omega \dot{x}_0' t^2 \sin\lambda$$

$$z' = z_0' + \dot{z}_0' t - \frac{1}{2} g t^2 + \omega \dot{x}_0' t^2 \cos\lambda$$

For this Problem $\lambda = 0$, $z_0' = 8000\,m$ $x_0' = y_0' = 0$

$\dot{x}_0' = \dot{z}_0' = 0$ $\quad \dot{y}_0' = 125\ m/s$

a) $z' = 8000 + \cancel{0} - \frac{1}{2} g t^2 + \cancel{0}$ hits ground when $z' = 0$

$$\frac{1}{2} g t^2 = 8000$$

$$t = \sqrt{\frac{16000}{9.8}} = \boxed{40.4\ s}$$

b) $y' = \cancel{0} + \dot{y}_0' t - \cancel{0} = (125\,m/s)(40.4\,s) = \boxed{5050.8\ m}$

c) $x' = 0 + 0 - \omega t^2\left[0 \overset{1}{\cos(0)} - 125 \overset{0}{\sin(0)}\right] + \frac{1}{3}\omega g t^3 \overset{1}{\cos(0)}$

$$x' = \frac{1}{3}(7.3\times10^{-5})(9.8)(40.4)^3 = \boxed{15.7\ m}$$

d) $x' = +\omega t^2 \dot{y}_0' \sin\lambda + \frac{1}{3}\omega g t^3 \cos\lambda = 0$

$$\tan\lambda = \frac{-\frac{1}{3}\cancel{\omega} g t^{\cancel{3}}}{\cancel{\omega}\cancel{t^2}\dot{y}_0'} = -\frac{tg}{3\dot{y}_0'}$$

$$\lambda = \mathrm{Tan}^{-1}\left(\frac{-gt}{3\dot{y}_0'}\right) = \mathrm{Tan}^{-1}\left(\frac{-9.8(40.4)}{3(125)}\right)$$

$$\lambda = \mathrm{Tan}^{-1}(-1.056) = \boxed{-46.55^\circ}$$