Homework Set #1

1. (F&C 1.3) Find the angle between the vectors $\mathbf{A} = a\hat{\mathbf{e}}_x + 2a\hat{\mathbf{e}}_y$ and $\mathbf{B} = a\hat{\mathbf{e}}_x + 2a\hat{\mathbf{e}}_y + 3a\hat{\mathbf{e}}_z$. (*Note:* These two vectors define a face diagonal and a body diagonal of a rectangular block of sides $a, 2a$ and $3a$. This type of calculation is often important in solid state physics.)

2. (F&C 1.3) For what value (or values) of q is the vector $\mathbf{A} = q\hat{\mathbf{e}}_x + 3\hat{\mathbf{e}}_y + \hat{\mathbf{e}}_z$ perpendicular to the vector $\mathbf{B} = q\hat{\mathbf{e}}_x - q\hat{\mathbf{e}}_y + 2\hat{\mathbf{e}}_z$?

3. (F&C 1.18) A buzzing fly moves in a helical path given by the equation

$$\mathbf{R} = b\sin(\omega t)\hat{\mathbf{e}}_x + b\cos(\omega t)\hat{\mathbf{e}}_y + ct^2\hat{\mathbf{e}}_z$$

where b, c and ω are constants. Show that the magnitude of the acceleration of the fly is constant.

4. (F&C 1.20) In cylindrical coordinates, the helical path of the fly from the previous problem is given by

$$\mathbf{R} = b\hat{\mathbf{e}}_r + ct^2\hat{\mathbf{e}}_z$$

with $\dot{\phi} = \omega$. Using cylindrical coordinates, show that the magnitude of the acceleration of the fly is constant and equal to that derived in the previous problem.

5. (F&C 1.19) A bee goes out from its hive in a spiral path given in cylindrical coordinates

$$r = be^{kt} \quad ; \quad \phi = \omega t \quad ; \quad z = 0$$

where b, k and ω are positive constants. Show that the angle between the velocity and acceleration vectors remains constant as the bee move outward and is given by $\theta = \arccos(\frac{k}{\sqrt{k^2+\omega^2}})$.

6. (F&C 1.8) Give and algebraic and a geometric proof of the following relation.

$$|\mathbf{A} + \mathbf{B}| \leq |\mathbf{A}| + |\mathbf{B}|$$

$$|\mathbf{A} \cdot \mathbf{B}| \leq |\mathbf{A}|\ |\mathbf{B}|$$

7. Show by direct differentiation of the position vector that the velocity and acceleration in spherical coordinates are given by

$$\mathbf{V} = \dot{r}\hat{\mathbf{e}}_r + r\dot{\phi}\sin\theta\hat{\mathbf{e}}_\phi + r\dot{\theta}\hat{\mathbf{e}}_\theta$$

and

$$\mathbf{A} = (\ddot{r} - r\dot{\phi}^2\sin^2\theta - r\dot{\theta}^2)\hat{\mathbf{e}}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta)\hat{\mathbf{e}}_\theta + (r\ddot{\phi}\sin\theta + 2r\dot{\phi}\dot{\theta}\cos\theta + 2\dot{r}\dot{\phi}\sin\theta)\hat{\mathbf{e}}_\phi$$

8. (F&C 1.23&1.24) Prove the following identities

$$\mathbf{v} \cdot \mathbf{a} = v\dot{v}$$

and

$$\frac{d}{dt}(\mathbf{r} \cdot (\mathbf{v} \times \mathbf{a})) = (\mathbf{r} \cdot (\mathbf{v} \times \dot{\mathbf{a}}))$$

9. Find the instantaneous velocity and acceleration of the bead depicted in both cylindrical and spherical coordinates.