

## Homework Set #2

1. (F&C 2.1) Find the velocity  $\dot{x}$  and the position  $x$  as functions of the time  $t$  for a particle of mass  $m$  which starts from rest at  $x = 0$  and  $t = 0$ , subject to the following force functions

(a)  $F_x = F_0 + ct$

(b)  $F_x = F_0 \sin(ct)$

(c)  $F_x = F_0 e^{ct}$

2. (F&C 2.2) Find the velocity  $\dot{x}$  as a function of the displacement  $x$  for a particle of mass  $m$  which starts from rest at  $x = 0$ , subject to the following force functions

(a)  $F_x = F_0 + cx$

(b)  $F_x = F_0 e^{-cx}$

(c)  $F_x = F_0 \cos(cx)$

3. (F&C 2.12 & 2.13) A particle is projected vertically upward in a constant gravitational field with an initial velocity  $v_0$ . Show that if there is a retarding force proportional to the square of the instantaneous velocity (i.e. quadratic drag), then the velocity of the particle when it returns to the ground is

$$v = \frac{v_0 v_t}{\sqrt{v_0^2 + v_t^2}}$$

where  $v_t = \sqrt{mg/c_2}$  is the terminal velocity of the particle.

4. An automobile driver traveling down an 8% grade slams on his breaks and skids 30 meters before hitting a parked car. A lawyer hires an expert who measures the coefficient of kinetic friction between the road and the tires to be  $\mu_k = 0.45$ . Is the lawyer correct to accuse the driver of exceeding the 25 MPH speed limit? Explain. (Assume that an 8% grade means that the road raises 8 meters vertically for every 100 meters traveled horizontally.)
5. (F&C 2.19) A boat with initial speed  $v_0$  is launched on a lake. The boat is slowed by the water with a retarding force given by  $F(v) = -\alpha e^{\beta v}$  where  $\alpha$  and  $\beta$  are positive constants. a) Find the speed of the boat as a function of time. b) Find the time required for the boat to stop. c) Find the distance required for the boat to stop.
6. (F&C 2.6) A particle of mass  $m$  moves along a frictionless, horizontal plane with a speed given by  $v(x) = \alpha/x$ , where  $x$  is the displacement from the origin and  $\alpha$  is a positive constant. Find the force  $F(x)$  to which the particle is subjected.

7. (F&C 2.15) Show that the terminal speed of a falling spherical object is given by

$$v_t = \sqrt{(mg/c_2) + (c_1/2c_2)^2} - (c_1/2c_2)$$

when *both* linear and quadratic terms in the drag force are taken into account.

8. A particle of mass  $m$  moves under the influence of a force given by

$$F = -kx + \frac{kx^3}{\alpha^2}$$

where  $k$  and  $\alpha$  are positive constants. a) Find the potential energy function  $U(x)$  (Take  $U(x=0) = 0$ ). b) Find any equilibrium points and determine if they are stable or unstable. c) Sketch the potential as a function of position. d) What is the maximum total energy,  $E_{max}$  a particle may have if its motion is to remain bounded? e) For the case of bounded motion with  $E < E_{max}$ , determine the turning points for the particle motion.