Homework Set #3

1. (F&C 3.3) A particle undergoes simple harmonic motion with a frequency of 10 Hz. Find the displacement x at any time t for the following initial conditions: at $t = 0$, $x = 0.25$m and $\dot{x} = 0.1$m/s.

2. (F&C 3.5) A particle undergoing simple harmonic motion has a velocity $\dot{x}_1$ when the displacement is x_1 and a velocity $\dot{x}_2$ when the displacement is x_2. Find the angular frequency and the amplitude of the motion in terms of the given quantities.

3. A body of uniform cross-sectional area A and mass density ρ floats is a liquid and at equilibrium displaces a volume V. Show that the period of small oscillations about the equilibrium position is given by

$$\tau = 2\pi\sqrt{V/gA}$$

   where g is the acceleration caused by gravity.(Hint: the two forces acting on the body are 1) gravity pulling it downwards and 2) the buoyant force (equal to the mass of the fluid displaced) pushing it upwards.)

4. Show that for a driven damped harmonic oscillator, the kinetic energy averaged over a complete period is given by

$$< T >= \frac{mA^2}{4}\frac{\omega^2}{(\omega_0^2 - \omega^2)^2 + 4\omega^2\gamma^2}$$

   At what frequency should the oscillator be driven in order to have the largest average kinetic energy.

5. (F&C 3.10) A damped harmonic oscillator with $m = 10$kg, $k = 250$N/m and $c = 60$kg/s is subject to a driving force given by $F_0 \cos(\omega t)$, where $F_0 = 48$N.

   (a) What value of ω results in steady-state oscillations with maximum amplitude? Under this condition:

   (b) What is the maximum amplitude?

   (c) What is the phase shift?

6. Find the general solution to the ordinary differential equation

$$\frac{d^2x}{dt^2} - 6\frac{dx}{dt} + 9x = 18t^2$$

7. Use the general solution $x(t)$ for the underdamped, critically damped and overdamped oscillator and solve for the constants of integration assuming that at $t = 0$, $x = x_0$ and $v = v_0$.

8. A mass m is attached to a spring (that obey's Hooke's Law) with spring constant k and is allowed to oscillate in a resistive medium where the resistance is proportional to the velocity of the mass and opposes the motion, i.e. $F_{drag} = -c\mathbf{v}$ (In all cases below, if I ask for a frequency, assume that I want angular frequency, ω.)

   (a) Assuming that $m = 0.01$ kg and $k = 1.44$ N/m, what is the natural frequency of the *undamped* oscillator.

   (b) At $t = 0$ the undamped oscillator is passing through the equilibrium point ($x = 0$) with a velocity of 3 m/s. What is the amplitude of the undamped oscillation?

   (c) Suppose the drag coefficient is measured to be 0.02 kg/s. What is the Quality factor of the damped oscillator? (You may use the weak damping result for this part.)

   (d) What is the frequency of the damped oscillator?

   (e) How long would we have to wait for the amplitude of the damped oscillations to decrease by a factor of two?

   (f) At what frequency should we *drive* the oscillator in order to get the maximum amplitude response?

   (g) What is the phase difference between the driver and the oscillator response if the oscillator is driven at ω =24 s^{-1}?