Homework Set #4

1. (F&C 4.1) Find the force for each of the following potential energy functions:
   (a) $U = cxyz + C$
   (b) $U = \alpha x^2 + \beta y^2 + \gamma z^2 + C$
   (c) $U = ce^{-(\alpha x + \beta y + \gamma z)}$
   (d) $U = cr^n$ (in spherical coordinates)

2. (F&C 4.2) By finding the curl, determine which of the following forces are conservative.
   (a) $\mathbf{F} = x\hat{\mathbf{e}}_x + y\hat{\mathbf{e}}_y + z\hat{\mathbf{e}}_z$
   (b) $\mathbf{F} = y\hat{\mathbf{e}}_x - x\hat{\mathbf{e}}_y + z^2\hat{\mathbf{e}}_z$
   (c) $\mathbf{F} = y\hat{\mathbf{e}}_x + x\hat{\mathbf{e}}_y + z^3\hat{\mathbf{e}}_z$

3. (F&C 4.3) Find the value of the constant c such that each of the following forces are conservative.
   (a) $\mathbf{F} = xy\hat{\mathbf{e}}_x + cx^2\hat{\mathbf{e}}_y + z^3\hat{\mathbf{e}}_z$
   (b) $\mathbf{F} = (z/y)\hat{\mathbf{e}}_x + (cxz/y^2)\hat{\mathbf{e}}_y + (x/y)\hat{\mathbf{e}}_z$

4. (F&C 4.8) A gun is located at the bottom of a hill that is inclined at an angle β with respect to the horizontal. If the gun is fired with an initial speed v_0 at an angle α with respect to the horizontal, show that the range of the gun measured up the slope of the hill is

   $$d = \frac{2v_0^2 \cos(\alpha)\ \sin(\alpha - \beta)}{g\ \cos^2(\beta)}$$

   Find the angle α at which the gun must be fired in order to obtain the maximum range and show that the maximum range is given by

   $$d_{max} = \frac{v_0^2}{g(1 + \sin(\beta))}$$

5. (F&C 4.9)A baseball pitcher can throw a ball horizontally more easily than vertically. Assume that the pitcher's throwing speed varies with elevation angle as $v_0 \cos(\alpha/2)$, where α is the initial throwing angle and v_0 is the initial speed if the ball is thrown horizontally. Neglecting air resistance, find the angle at which the ball must be thrown in order to achieve maximum height and maximum range. (Once you have an equation for the Range and the height of the ball you may find the maximum values numerically. I found it to be a whole lot easier.)

6. In class we showed that when linear drag is included in the equations for a projectile that is lauched from $x = y = 0$ with a speed v_0 at anlge α above the horizonatl the x and y position are given as a function of time by

$$x(t) = \frac{U}{\kappa}\left[1 - e^{-\kappa t}\right]$$

and

$$y(t) = -\frac{g}{\kappa}t + \frac{\kappa V + g}{\kappa^2}\left[1 - e^{-\kappa t}\right]$$

where $U = v_0 \cos\alpha$ and $V = v_0 \sin\alpha$ and c_1/m.

   (a) In the limit of weak damping (i.e. $\kappa \to 0$) show that to first order in κ, the time (T) when the particle hits the ground is given by

   $$T = \frac{2V/g}{1 + \kappa V/g} + \frac{1}{3}\kappa T^2$$

   (Hint: you must keep terms to third order in κ in your expansion of $e^{-\kappa T}$. Also note that we have not fully solved this equation for T since it appears on both sides of the equation.)

   (b) In the limit of $\kappa \to 0$ show that $T(\kappa \to 0) = T_0 = 2V/g$.

   (c) Using the result of parts a and b show that the time the partilce is in the air is given by

   $$T = \frac{2V}{g}\left[1 - \frac{\kappa V}{3g}\right] = T_0\left[1 - \frac{\kappa T_0}{6}\right]$$

7. Using the result of the previous problem show that

   (a) the range $R = x(T)$ is given by

   $$R = \frac{2UV}{g}\left[1 - \frac{4\kappa V}{3g}\right]$$

   and that

   (b) the magnitude in the reduction of the range as compared to the zero drag case is given by

   $$\Delta R = \frac{4\kappa v_0^3}{3g^2}\sin(\alpha)\sin(2\alpha)$$

8. (F&C 4.9) A particle is placed on a smooth sphere of radius b at a distance of $b/2$ above the central plane. As the particle slides down the side of the sphere, at what point does it leave the surface?