

Homework Set #6

1. Assume that the earth's orbit is circular and that the sun's mass suddenly decreases by a factor of two (a highly unlikely event). What orbit does the earth then have? Does the earth escape the solar system?
2. (F&C 6.10) Find the force law for a central force that allows a particle to move in a spiral orbit given by $r = r_0 e^{k\theta}$ where r_0 and k are constants. Also find θ and r as functions of time.
3. (F&C 6.12) The orbit of a particle moving in a central field is a circle passing through the origin, i.e. $r = r_0 \cos \theta$. Show that the force law is an inverse-fifth power.
4. (F&C 6.24) Show that the stability condition for a circular orbit of radius a , i.e.

$$f(a) + \frac{a}{3} \left[\frac{df}{dr} \right]_{r=a} < 0$$

is equivalent to the condition $d^2V(r)/dr^2 > 0$ for $r = a$ where $V(r)$ is the effective potential given by $V(r) = U(r) + m\ell^2/2r^2$.

5. (F&C 6.16) Given that the perihelion of Halley's comet is 55,000,000 mi and its eccentricity is 0.967, find the period the comet. Also find its speed at perihelion and aphelion.
6. (F&C 6.27) A comet is going in a parabolic orbit lying in the plane of the earth's orbit. Assuming that the earth's orbit is a circle of radius a , show that the points where the comets orbits intersects the earth's orbit are given by

$$\cos \theta_c = -1 + 2p/a$$

where p is the perihelion distance of the comet defined as $\theta = 0$.

7. (F&C 6.28) Use the result from the previous problem to show that the time that the comet remains within the earth's orbit is

$$\frac{2^{1/2}}{3\pi} \left(\frac{2p}{a} + 1 \right) \left(1 - \frac{p}{a} \right)^{1/2} \tau_E$$

where τ_E is the period of the earth's orbit (i.e. 1 year). Use this to calculate the time that Halley's comet remains inside the earth's orbit. (Note: this is not strictly correct since Halley's comet follows an elliptical path, but its eccentricity is so close to unity that it gives a reasonable approximation.)

8. A space craft is parked in a circular orbit 200 km above the surface of the earth. We want to use a Hohmann transfer to send the space craft to the moon's orbit. What are the total change in velocity Δv and the transfer time required?