Homework Set #7

1. (F&C 7.11)Show that the angular momentum of a two-particle system is

$$\mathbf{L} = \mathbf{r}_{cm} \times M\mathbf{v}_{cm} + \mathbf{R} \times \mu\mathbf{V}$$

   where $M = m_1 + m_2$, μ is the reduce mass, $\mathbf{R}$ is the relative position vector and $\mathbf{V}$ is the relative velocity vector of the two particles.

2. (F&C 7.8)Show that the kinetic energy of a two-particle system is

$$T = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}\mu V^2$$

   where $M = m_1 + m_2$, μ is the reduce mass and V is the relative speed of the two particles.

3. (F&C 7.5) An artillery shell is fired at an angle of elevation of 60° with an initial speed v_0. At the uppermost part of its trajectory, the shell bursts into two equal fragments, one of which moves directly upward relative to the ground with an initial speed $v_0/2$. What is the speed and direction of the other fragment immediately after the burst. Neglect drag.

4. In homework set #6 we found that in order to take a satellite from a low earth circular orbit into an elliptical orbit that intercepts the orbit of moon, we had to increase it velocity by 3.13×10^3m/s. If the exhaust velocity of the rocket is 2.60×10^3m/s (representative of the Apollo program), what percentage of the rockets mass must be made up of fuel. (Note that this does not include the fuel that must remain to put the rocket into lunar orbit or the fuel needed to get back.)

5. Consider a rocket making a vertical ascent in a uniform gravitational field, g. Show that the height of the rocket when the fuel is exhausted is given by

$$x = ut_b - \frac{1}{2}gt_b^2 - \frac{um}{\alpha}\ln(m_0/m)$$

   where u is the exhaust velocity of the fuel, t_b is the time at burnout, α is the fuel burn rate and m_0 (m) is the intial (final) mass of the rocket.

6. A rocket is making its way through a giant dust cloud that procuces a linear drag force $F_{drag} = -cv$ on the rocket. Neglecting gravity and assuming that the rocket starts with an initial speed $v_0$inside the dust cloud, find the speed of the rocket after it has used up its fuel. Use the standard definitions from class, i.e. u is the exhaust velocity of the fuel, α is the fuel burn rate and m_0 (m) is the intial (final) mass of the rocket.

7. (F&C 8.3) A solid uniform sphere of radius a has a spherical cavity of radius $a/2$ centered at a point $a/2$ from the center of the sphere. Find the center of mass of the sphere.

8. Find the moment of inertia of a uniform rod of length a about an arbitrary point x_0 somewhere along the length of the rod. Show that if $x_0 = 0$ (i.e at the end of the rod) then $I = ma^2/3$ and if $x_0 = a/2$ (i.e the center of the rod) then $I = ma^2/12$.

9. Consider a star to be a uniform spheical body of total mass M and radius R_0. The star is intially rotating with angular velocity ω_0. If the star suddenly collapses to 1/1000 of its initial radius without changing mass, (a fairly poor approximation of collapsing into a neutrn star) what is the new frequency of rotaion?