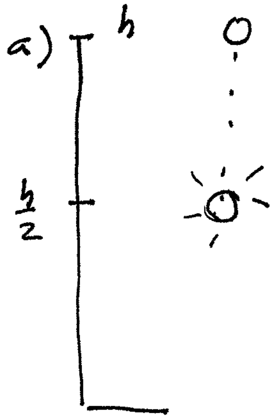
1) a)



use conservation of energy

$$mgh = mg\frac{h}{2} + \frac{1}{2}mv^2$$

$$\cancel{m}g\frac{h}{2} = \frac{1}{\cancel{2}}\cancel{m}v^2$$

$$\boxed{v = \sqrt{gh} \quad \text{down}}$$

or $\boxed{v = -\sqrt{gh}\ \hat{e}_y}$

b) Momentum must be conserved.

- x-momentum before explosion = 0
- after explosion, feet & head momenta cancel.

$$\frac{M}{3}V_2\hat{e}_y = -M\sqrt{gh}\ \hat{e}_y$$

$$\boxed{V_2 = -3\sqrt{gh}\ \hat{e}_y}$$

c)

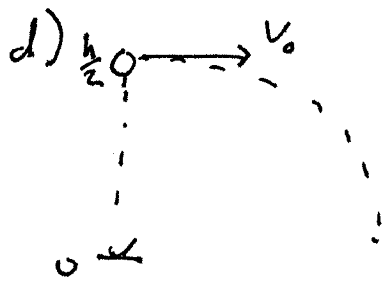
$$KE_i = \frac{1}{2}M(gh)$$

$$KE_f = \frac{1}{2}\left(\frac{M}{3}\right)V_0^2 + \frac{1}{2}\left(\frac{M}{3}\right)v_0^2 + \frac{1}{2}\left(\frac{M}{3}\right)(9gh)$$

$$= \frac{1}{3}MV_0^2 + \frac{3}{2}Mgh$$

$$\Delta KE = KE_f - KE_i = \boxed{\frac{1}{3}MV_0^2 + \frac{1}{2}Mgh = \Delta KE}$$

d)



$$V_x = V_0$$

$$mg\frac{h}{\cancel{2}} = \frac{1}{\cancel{2}}mv_y^2$$

$$v_y = \sqrt{gh}$$

$$\boxed{\underline{V}_h = V_0\hat{e}_x - \sqrt{gh}\ \hat{e}_y}$$

2) a) $m \dfrac{dv}{dt} = -cv^{1/2}$

b) $mv \dfrac{dv}{dx} = -cv^{1/2}$ $\qquad \gamma = \frac{c}{m}$

c) $\int_{v_0}^{v} v^{1/2}\,dv = -\frac{c}{m}\,dx = -\gamma \int_0^{x} dx$

$\frac{1}{\frac{1}{2}+1} v^{\frac{1}{2}+1}\Big|_{v_0}^{v} = \frac{1}{3/2} v^{3/2}\Big|_{v_0}^{v} = \frac{2}{3} v^{3/2} - \frac{2}{3} v_0^{3/2} = -\gamma x$

$v^{3/2} = v_0^{3/2} - \frac{3}{2}\gamma x$

Max dist when $v = 0$

$v_0^{3/2} = \frac{3}{2}\gamma x_M$

$\boxed{x_M = \frac{2}{3\gamma} v_0^{3/2}}$

d) Let $v_0 \to 2v_0$

$x_M' = \frac{2}{3\gamma}(2)^{3/2} v_0^{3/2}$

$= 2^{3/2} x_M \Rightarrow$ it goes $2^{3/2}$ times farther.

e) $\int_{v_0}^{0} v^{-1/2}\,dv = -\gamma \int_0^{t} dt$

$\frac{1}{-\frac{1}{2}+1} v^{-\frac{1}{2}+1}\Big|_{v_0}^{0} = -\gamma t$

$2v^{1/2}\Big|_{v_0}^{0} = -2v_0^{1/2} = -\gamma t$

$\boxed{t_{max} = \frac{2v_0^{1/2}}{\gamma}}$

3) a) $r = \alpha\theta \qquad u = \frac{1}{r} = \frac{1}{\alpha\theta} = \frac{\theta^{-1}}{\alpha}$

$$\frac{du}{d\theta} = -\frac{\theta^{-2}}{\alpha} \;;\quad \frac{d^2u}{d\theta^2} = +2\frac{\theta^{-3}}{\alpha} = \frac{2\alpha^2}{\alpha^3\theta^3} = 2\alpha^2u^3$$

$$\frac{d^2u}{d\theta^2} + u = -\frac{1}{m\ell^2u^2} f(u)$$

$$-(2\alpha^2u^3 + u)(m\ell^2u^2) = f(u) = -m\ell^2\left[2\alpha^2u^5 + u^3\right]$$

$$\boxed{f(r) = -m\ell^2\left[\frac{2\alpha^2}{r^5} + \frac{1}{r^3}\right]}$$

b) Orbits for $f(r) = -kr^n$ are only stable if $n > -3$.
Both terms have $n \le -3 \Rightarrow$ unstable.

c) $\ell = r^2\dot\theta \Rightarrow \frac{d\theta}{dt} = \frac{\ell}{r^2} = \frac{\ell}{\alpha^2\theta^2} = \frac{d\theta}{dt}$

$$\int_0^\theta \theta^2 d\theta = \frac{\ell}{\alpha^2}\int_0^t dt$$

$$\frac{1}{3}\theta^3 = \frac{\ell}{\alpha^2}t \quad\Rightarrow \boxed{\theta = \left(\frac{3\ell}{\alpha^2}t\right)^{1/3}}$$

d) $r = \alpha\theta = \alpha\left(\frac{3\ell}{\alpha^2}t\right)^{1/3}$

$$\boxed{r = (3\alpha\ell t)^{1/3}}$$

4) a) $m\ddot{x} + c\dot{x} + kx = 0$

or

$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = 0 \qquad 2\gamma = c/m \; ; \; \omega_0^2 = \frac{k}{m}$

b) $\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{8000}{20}} = \sqrt{400} = \boxed{20\,s^{-1}}$

c) $\frac{1}{2}kA^2 = \frac{1}{2}Mv^2$ (cons. of energy)

$A^2 = \frac{M}{k}v^2 = \frac{v^2}{\omega_0^2} \Rightarrow A = \frac{v}{\omega_0} = \frac{5\,m/s}{20\,s^{-1}} = \boxed{\frac{1}{4}\,m}$

d) $x(t) = A\sin(\omega_0 t + \phi)$

at $t = 0$, $x = 0 = A\sin\phi \Rightarrow \phi = 0$

$A = \frac{1}{4}\,m$

$\boxed{x = \frac{1}{4}\sin(20t)}$

e) $c = 40\,kg/s \Rightarrow 2\gamma = \frac{c}{m} = \frac{40\,kg/s}{20\,kg} = 2\,s^{-1}$ $\boxed{\gamma = 1}$

$Q \approx \frac{\omega_0}{2\gamma} = \frac{20\,s^{-1}}{2\,s^{-1}} = \boxed{10 = Q}$

f) $\omega_d = \sqrt{\omega_0^2 - \gamma^2}$

$\omega_d = \sqrt{20^2 - 1} = \sqrt{399}\,s^{-1} = \boxed{19.97\,s^{-1}}$

g) $e^{-\gamma\tau} = 0.1 \Rightarrow \tau = -\frac{1}{\gamma}\ln(0.1) = -\ln(0.1)$

$\boxed{\tau = 2.3\,s}$

h) For critical damping $\gamma = \omega_0 \; ; \; \frac{c}{2m} = \omega_0$

$c = 2m\omega_0 = 2(20\,kg)(20\,s^{-1})$

$\boxed{C_{crit} = 800\,kg/s}$

i) $\omega_R = \sqrt{\omega_0^2 - 2\gamma^2} = \sqrt{398} = \boxed{19.95\,s^{-1}}$

5) $x' = x_0' + v_{x0}' t - \omega t^2 [v_{z0}' \cos\lambda - v_{y0}' \sin\lambda] + \frac{1}{3}\omega g t^3 \cos\lambda$

$y' = y_0' + v_{y0}' t - \omega t^2 v_{x0}' \sin\lambda$

$z' = z_0' + v_{z0}' t - \frac{1}{2} g t^2 + \omega t^2 v_{x0}' \cos\lambda$

take

$\lambda = 0$

$z_0' = h \quad x_0' = 0 \quad y_0' = 0 \qquad h = 8000 m$

$v_{x0}' = 0 \quad v_{y0}' = v_0 \quad v_{z0}' = 0 \qquad v_0 = 125 m/s$

a) $0 = h - \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(8000 m)}{9.8 m/s^2}} =$

$\boxed{t = 40.4 s}$

b) $y' = 0 + v_0 t = (125 m/s)(40.4 s) = \boxed{5050 m}$

c) $x' = + \omega t^2 v_0 \cancel{\sin\lambda} + \frac{1}{3}\omega g t^3 \cos\lambda$

$= \frac{1}{3}(7.3 \times 10^{-5} s^{-1})(9.8 m/s^2)(40.4 s)^3$

$= \boxed{15.66 m}$

d) to cancel out $(\lambda \neq 0)$

$\cancel{\omega} t^{\cancel{2}} v_0 \sin\lambda = -\frac{1}{3}\cancel{\omega} g t^{\cancel{3}} \cos\lambda$

$\tan\lambda = -\frac{g t}{3 v_0} = -\frac{(9.8 m/s^2)(40.4 s)}{3 \ 125 m/s} = -1.06$

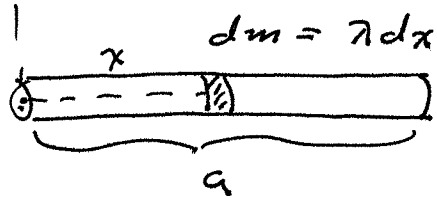
$\lambda = \tan^{-1}(-1.06) = -0.8125$ rad

$\lambda = -39.09°$

XC

$dm = \lambda dx \qquad M = \lambda a$

a)



$$I = \int_0^a x^2 \lambda dx = \frac{1}{3}\lambda a^3 = \frac{1}{3} M a^2$$

$$L_0 = I_0 \omega_0 = \left(\frac{1}{3} M a^2\right) \omega_0$$

$$I_{sph} = \frac{2}{5} M r^2 = \frac{2}{5}(2M)(2a)^2$$

$$= \frac{4}{5} M (4a^2) = \frac{16}{5} M a^2$$

$$I_{tot} = I_1 + I_2 = \left(\frac{1}{3} M a^2\right) + \frac{16}{5} M a^2$$

$$= \left(\frac{5}{15} + \frac{48}{15}\right) M a^2 = \left(\frac{53}{15} M a^2\right)$$

$$L_f = I_{tot}\,\omega = I_0 \omega_0$$

$$\frac{53}{15} \cancel{Ma^2}\, \omega = \left(\frac{1}{3} \cancel{Ma^2}\right) \omega_0$$

$$\omega = \left(\frac{15}{53 \cdot 3}\right) \omega_0 = \boxed{\frac{5}{53}\omega_0}$$