1) $A = a\hat{e}_x + 2a\hat{e}_y$

$|A| = \sqrt{a^2 + 4a^2} = \sqrt{5}\,a$

$B = a\hat{e}_x + 2a\hat{e}_y + 3a\hat{e}_z$

$|B| = \sqrt{a^2 + 4a^2 + 9a^2} = \sqrt{14}\,a$

$\cos\theta = \dfrac{A \cdot B}{|A||B|}$

$\cos\theta = \dfrac{a^2 + 4a^2}{(\sqrt{5}a)(\sqrt{14}a)} = \dfrac{5}{\sqrt{5}\sqrt{14}} = \sqrt{\dfrac{5}{14}}$

$\boxed{\theta = 53.3^\circ}$

2) If $A \perp B \Rightarrow A \cdot B = 0$

$A = q\, e_x + 3 e_y + e_z$

$B = q\, e_x - q\,\hat{e}_y + 2\hat{e}_z$

$A \cdot B = q^2 - 3q + 2 = 0$

$(q-1)(q-2) = 0$

$\boxed{q = 1 \text{ or } 2}$

3) $R = b\sin(\omega t)\hat{e}_x + b\cos(\omega t\hat{e}_y) + ct^2\hat{e}_z$

$V = +\omega b\cos(\omega t)\hat{e}_x - \omega b\sin(\omega t)\hat{e}_y + 2ct\,\hat{e}_z$

$a = -\omega^2 b\sin(\omega t)\,\hat{e}_x - \omega^2 b\cos(\omega t)\hat{e}_y + 2c\,\hat{e}_z$

$|a| = \left[\omega^4 b^2 \sin^2(\omega t) + \omega^4 b^2\cos^2(\omega t) + 4c^2\right]^{1/2}$

$\boxed{|a| = \sqrt{\omega^4 b^2 + 4c^2}}$

since indep of time
$\Rightarrow |a| = $ const.

4) $R = r\hat{e}_r + z\hat{e}_z$

$= b\hat{e}_r + ct^2\hat{e}_z$

$V = \dot{r}\hat{e}_r + r\dot{\phi}\,\hat{e}_\phi + \dot{z}\hat{e}_z$

$= b\omega\,\hat{e}_\phi + 2ct\,\hat{e}_z$

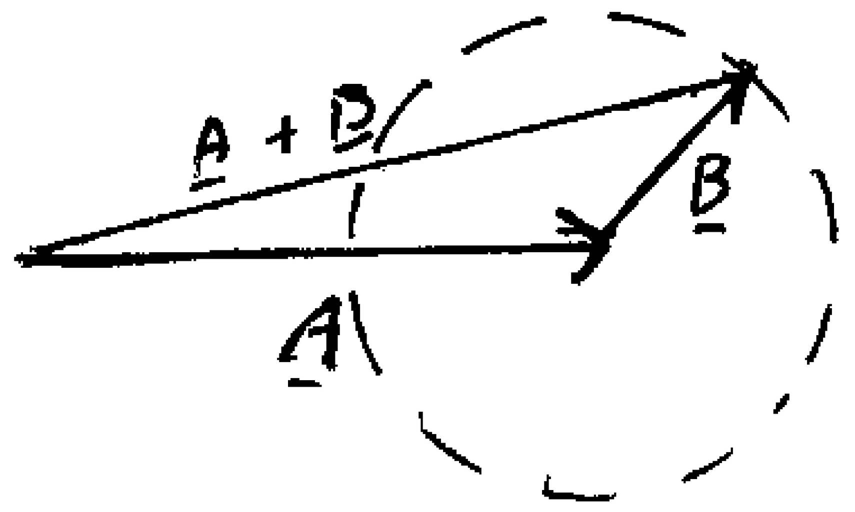
$a = (\ddot{r} - r\dot{\phi}^2)\hat{e}_r + (r\ddot{\phi} + 2\dot{r}\dot{\phi})\hat{e}_\phi + \ddot{z}\hat{e}_z$

$= -b\omega^2\hat{e}_r + 2c\,\hat{e}_z$

$\boxed{|a| = \sqrt{b^2\omega^4 + 4c^2}}$

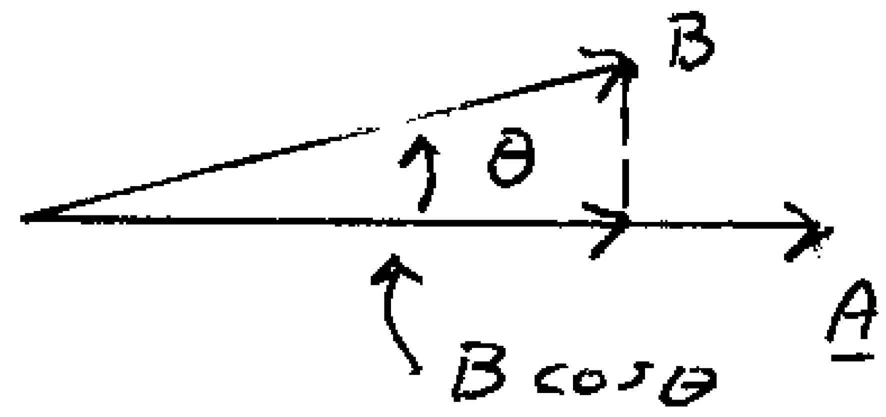
same as Prob 3

6) (cont)



A vector of $|\underline{B}|$ when added to $\underline{A}$ will fall on circle. Max of $|A|+|B|$ obviously when $\underline{A} \parallel \underline{B}$.

b) $$|\underline{A} \cdot \underline{B}| = |A||B| \cos\theta \leq |\underline{A}||\underline{B}|$$



7) $\underline{R} = r\hat{e}_r$ Given
$$\dot{\hat{e}}_r = \dot{\theta}\hat{e}_\theta + \dot{\phi}\sin\theta\,\hat{e}_\phi$$
$$\dot{\hat{e}}_\theta = -\dot{\theta}\hat{e}_r + \dot{\phi}\cos\theta\,\hat{e}_\phi$$
$$\dot{\hat{e}}_\phi = -\dot{\phi}\sin\theta\,\hat{e}_r - \dot{\phi}\cos\theta\,\hat{e}_\theta$$

$$\underline{V} = \dot{\underline{R}} = \dot{r}\hat{e}_r + r\dot{\hat{e}}_r$$
$$= \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta + r\dot{\phi}\sin\theta\,\hat{e}_\phi$$

$$\underline{a} = \dot{\underline{V}} = \left[\ddot{r}\hat{e}_r + \dot{r}\dot{\hat{e}}_r\right] + \left[\dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\dot{\hat{e}}_\theta\right]$$
$$+\left[\dot{r}\dot{\phi}\sin\theta\,\hat{e}_\phi + r\ddot{\phi}\sin\theta\,\hat{e}_\phi + r\dot{\phi}\dot{\theta}\cos\theta\,\hat{e}_\phi + r\dot{\phi}\sin\theta\,\dot{\hat{e}}_\phi\right]$$

$$= \left[\ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + \dot{r}\dot{\phi}\sin\theta\,\hat{e}_\phi\right] + \left[\dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta - r\dot{\theta}^2\hat{e}_r\right.$$
$$\left. + r\dot{\phi}\dot{\theta}\cos\theta\,\hat{e}_\phi\right] + \left[\dot{r}\dot{\phi}\sin\theta\,\hat{e}_\phi + r\ddot{\phi}\sin\theta\,\hat{e}_\phi + r\dot{\phi}\dot{\theta}\cos\theta\,\hat{e}_\phi\right.$$
$$\left. - r\dot{\phi}^2\sin^2\theta\,\hat{e}_r - r\dot{\phi}^2\sin\theta\cos\theta\,\hat{e}_\theta\right]$$

$$= \hat{e}_r\left[\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta\right] + \hat{e}_\theta\left[r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta\right]$$
$$+ \hat{e}_\phi\left[r\ddot{\phi}\sin\theta + 2\dot{r}\dot{\phi}\sin\theta + 2r\dot{\phi}\dot{\theta}\cos\theta\right]$$

8 a) $$\frac{d}{dt}(\underline{v}\cdot\underline{v}) = 2\underline{v}\cdot\underline{a}$$
$$= \frac{d}{dt}(v^2) = 2v\dot{v}$$
$(v = |\underline{v}|)$ $\Rightarrow$
$$\underline{v}\cdot\underline{a} = v\dot{v}$$

b) $$\frac{d}{dt}(\underline{r}\cdot(\underline{v}\times\underline{a})) = \dot{\underline{r}}\cdot(\underline{v}\times\underline{a})$$
$$+ \underline{r}\cdot(\dot{\underline{v}}\times\underline{a}) + \underline{r}\cdot(\underline{v}\times\dot{\underline{a}})$$
$$= \underbrace{\underline{v}\cdot(\underline{v}\times\underline{a})}_{=0} + \underbrace{\underline{r}\cdot(\underline{a}\times\underline{a})}_{=0} + \underline{r}\cdot(\underline{v}\times\dot{\underline{a}})$$
$(\underline{v} \perp (\underline{v}\times\underline{a}))$
$$= \underline{r}\cdot(\underline{v}\times\dot{\underline{a}})$$

5) $\underline{R} = r\hat{e}_r$

$\underline{V} = \dot{r}\hat{e}_r + r\dot{\phi}\hat{e}_\phi$

$\underline{a} = (\ddot{r} - r\dot{\phi}^2)\hat{e}_r + (r\ddot{\phi} + 2\dot{r}\dot{\phi})\hat{e}_\phi$

$r = be^{kt}$

$\dot{r} = kbe^{kt} = kr$

$\ddot{r} = k^2be^{kt} = k^2r$

$\phi = \omega t$

$\dot{\phi} = \omega$

$\underline{V} = kr\,\hat{e}_r + \omega r\,\hat{e}_\phi$

$|\underline{V}| = \sqrt{k^2+\omega^2}\; r$

$\underline{a} = (k^2-\omega^2)r\,\hat{e}_r + 2k\omega r\,\hat{e}_\phi$

$$|a| = \left[(k^2-\omega^2)^2 r^2 + 4k^2\omega^2 r^2\right]^{1/2}$$

$$= \left[k^4 - 2k^2\omega^2 + \omega^4 + 4k^2\omega^2\right]^{1/2} r$$

$$= \left[k^4 + 2k^2\omega^2 + \omega^4\right]^{1/2} r$$

$$= \left[(k^2+\omega^2)^2\right]^{1/2} r$$

$$= (k^2+\omega^2)\, r$$

$$\cos\theta = \frac{a\cdot V}{|a|\,|V|} = \frac{k(k^2-\omega^2)\cancel{r^2} + 2k\omega^2\cancel{r^2}}{\sqrt{k^2+\omega^2}\,[k^2+\omega^2]\,\cancel{r^2}}$$

$$= \frac{k\,[k^2-\omega^2+2\omega^2]}{\sqrt{k^2+\omega^2}\,[k^2+\omega^2]} = \frac{k\,[\cancel{k^2+\omega^2}]}{\sqrt{k^2+\omega^2}\,[\cancel{k^2+\omega^2}]}$$

OR

$$\boxed{\cos\theta = \frac{k}{\sqrt{k^2+\omega^2}}} = \text{const} \Rightarrow \theta = \text{const}.$$

---

6) $$|\underline{A}+\underline{B}| = \left[(\underline{A}+\underline{B})\cdot(\underline{A}+\underline{B})\right]^{1/2} = \left[\underline{A}^2 + \underline{B}^2 + 2\underline{A}\cdot\underline{B}\right]^{1/2}$$

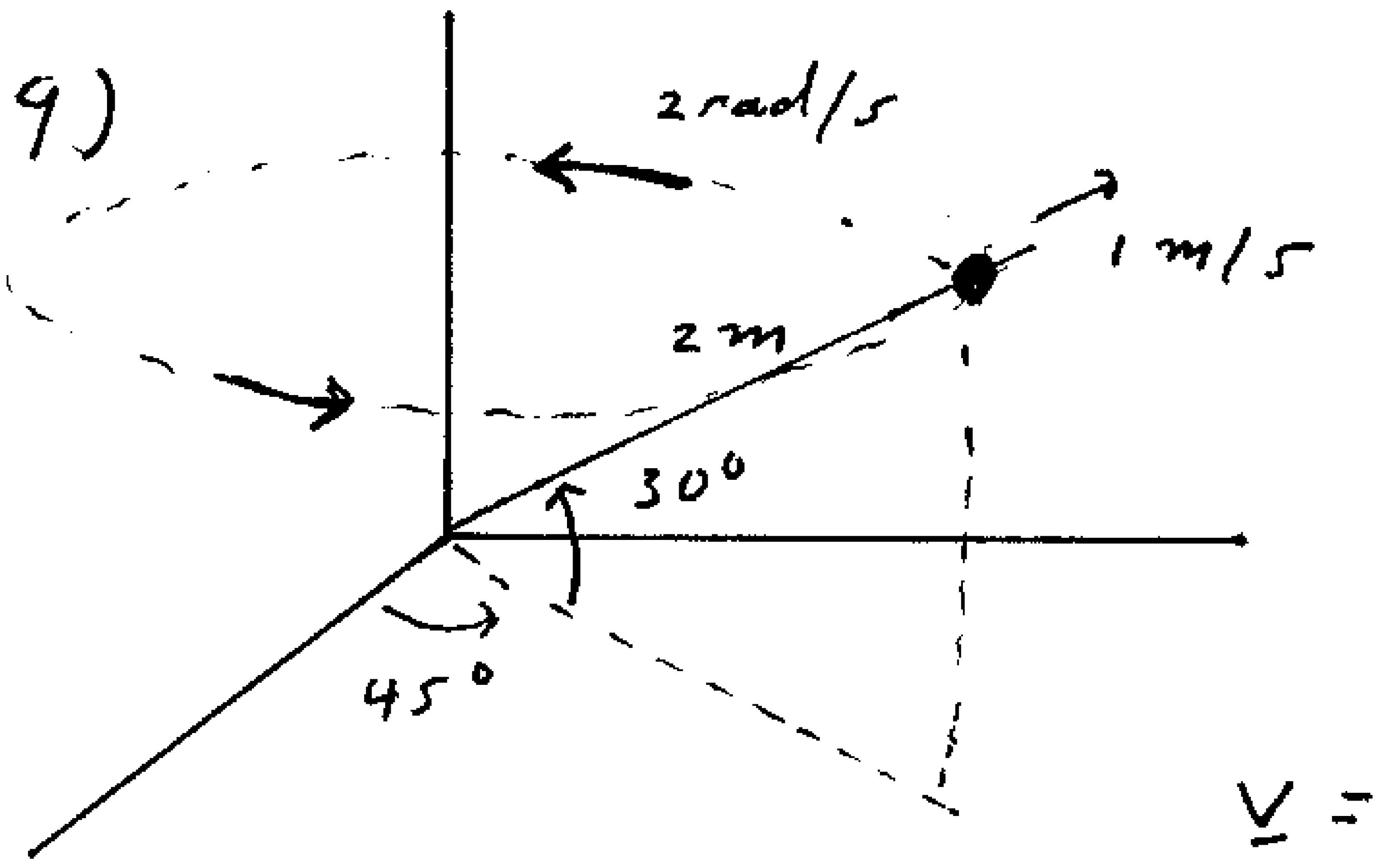
$$= \left[\underline{A}^2 + \underline{B}^2 + 2|A||B|\cos\theta\right]^{1/2}$$

$$\leq \left[\underline{A}^2 + \underline{B}^2 + 2|A||B|\right]^{1/2} = \left[(|A|+|B|)^2\right]^{1/2}$$

$$= |A| + |B|$$

$$\Rightarrow |\underline{A}+\underline{B}| \leq |\underline{A}| + |\underline{B}|$$

9)



spherical

$r = 2m \quad \theta = 60° \quad \phi = 45°$

$\dot{r} = 1 m/s \quad \dot{\theta} = 0 \quad \dot{\phi} = 2\ rad/s$

$\ddot{r} = 0 \quad \ddot{\theta} = 0 \quad \ddot{\phi} = 0$

$$\underline{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta + r\sin\theta\,\dot{\phi}\,\hat{e}_\phi$$

$$= 1 m/s\ \hat{e}_r + 2(0)\hat{e}_\theta + 2m\sin(60°)\,2s^{-1}\,\hat{e}_\phi$$

$$= 1 m/s\ \hat{e}_r + 3.46\ \hat{e}_\phi$$

$$= 1 m/s\ \hat{e}_r + 2\sqrt{3}\ \hat{e}_\phi$$

$$\underline{a} = (\cancel{\ddot{r}} - r\dot{\phi}^2\sin^2\theta - \cancel{r\dot{\theta}^2})\hat{e}_r + [\cancel{r\ddot{\theta}} + \cancel{2\dot{r}\dot{\theta}} - r\dot{\phi}^2\sin\theta\cos\theta]\hat{e}_\theta$$
$$+ \hat{e}_\phi\,[\cancel{r\ddot{\phi}\sin\theta} + 2\dot{r}\dot{\phi}\sin\theta + \cancel{2r\dot{\phi}\dot{\theta}\cos\theta}]$$

$$= -r\dot{\phi}^2\sin^2\theta\ \hat{e}_r - r\dot{\phi}^2\sin\theta\cos\theta\ \hat{e}_\theta + 2\dot{r}\dot{\phi}\sin\theta\ \hat{e}_\phi$$

$$= -(2m)(2s^{-1})^2\sin^2(60)\hat{e}_r - 2m(2s^{-1})^2\sin(60)\cos 60\,\hat{e}_\theta + 2(1m/s)(2s^{-1})\sin(60°)\hat{e}_\phi$$

$$= -6\ m/s^2\ \hat{e}_r - 2\sqrt{3}\ m/s^2\hat{e}_\theta + 2\sqrt{3}\ m/s^2\ \hat{e}_\phi$$
$$= (-6\hat{e}_r - 3.46\ \hat{e}_\theta + 3.46\ \hat{e}_\phi)\ m/s$$

cylindrical

$r = 2m\cos(30°) = \sqrt{3}\ m \quad \phi = 45° \quad z = 2m\sin(30°) = 1m$

$\dot{r} = 1m/s\cos 30°) = \frac{\sqrt{3}}{2}\ m/s \quad \dot{\phi} = 2s^{-1} \quad \dot{z} = 1m/s\sin 30° = 0.5\ m/s$

$\ddot{r} = 0 \quad \ddot{\phi} = 0 \quad \ddot{z} = 0$

$$\underline{v} = \dot{r}\,\hat{e}_r + r\dot{\phi}\,\hat{e}_\phi + \dot{z}\,\hat{e}_z$$
$$= \left(\frac{\sqrt{3}}{2}\,\hat{e}_r + \sqrt{3}\,2\hat{e}_\phi + \frac{1}{2}\,e_z\right) m/s = (0.866\ \hat{e}_r + 3.46\,\hat{e}_\phi + 0.5\,\hat{e}_z)\ m/s$$

$$\underline{a} = (\cancel{\ddot{r}} - r\dot{\phi}^2)\hat{e}_r + (\cancel{r\ddot{\phi}} + 2\dot{r}\dot{\phi}\,\hat{e}_\phi) + \ddot{z}\,\hat{e}_z$$

$$= -\sqrt{3}\,2^2\,\frac{m}{s^2}\hat{e}_r + 2\left(\frac{\sqrt{3}}{2}\right)2\ m/s^2\hat{e}_\phi + 0 \quad = (-4\sqrt{3}\ \hat{e}_r + 2\sqrt{3}\,\hat{e}_\phi)\ m/s^2$$
$$= (6.93\ \hat{e}_r + 3.46\ \hat{e}_\phi)\ m/s^2$$