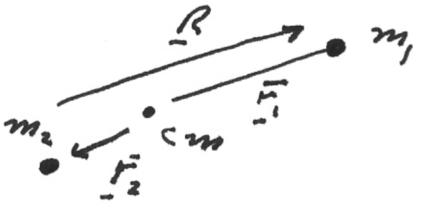


1) $L = \underline{r}_{cm} \times M v_{cm} + \sum_i \bar{\underline{r}}_i \times m_i \bar{\underline{v}}_i$

$$\underline{R} = \bar{\underline{r}}_1 - \bar{\underline{r}}_2$$

$$\underline{V} = \dot{\bar{\underline{r}}}_1 - \dot{\bar{\underline{r}}}_2 = \bar{\underline{v}}_1 - \bar{\underline{v}}_2$$

$$m_1 \bar{\underline{r}}_1 + m_2 \bar{\underline{r}}_2 = 0$$

Consider the 2nd term on R.H.S

$$\bar{\underline{L}} = \sum_i \bar{\underline{r}}_i \times m_i \bar{\underline{v}}_i = \bar{\underline{r}}_1 \times m_1 \bar{\underline{v}}_1 + \bar{\underline{r}}_2 \times m_2 \bar{\underline{v}}_2$$

But $m_1 \bar{\underline{r}}_1 = -m_2 \bar{\underline{r}}_2 \Rightarrow \bar{\underline{r}}_2 = -\frac{m_1}{m_2} \bar{\underline{r}}_1$

$$\bar{\underline{v}}_2 = -\frac{m_1}{m_2} \bar{\underline{v}}_1$$

$$\bar{\underline{L}} = (\bar{\underline{r}}_1 \times m_1 \bar{\underline{v}}_1) + -\left(\frac{m_1}{m_2}\right) \bar{\underline{r}}_1 \times \cancel{m_2}\left(-\frac{m_1}{\cancel{m_2}}\right) \bar{\underline{v}}_1$$

$$= (\bar{\underline{r}}_1 \times m_1 \bar{\underline{v}}_1) + \frac{m_1}{m_2}(\bar{\underline{r}}_1 \times m_1 \bar{\underline{v}}_1) = \left(1 + \frac{m_1}{m_2}\right)(\bar{\underline{r}}_1 \times m_1 \bar{\underline{v}}_1)$$

But $\underline{R} = \bar{\underline{r}}_1 - \bar{\underline{r}}_2 = \bar{\underline{r}}_1 \left(1 + \frac{m_1}{m_2}\right)$

$$\underline{V} = \bar{\underline{v}}_1 - \bar{\underline{v}}_2 = \bar{\underline{v}}_1 \left(1 + \frac{m_1}{m_2}\right)$$

$$\bar{\underline{L}} = \frac{m_1}{1 + m_1/m_2} (\underline{R} \times \underline{V}) = \frac{m_1 m_2}{m_1 + m_2} (\underline{R} \times \bar{\underline{V}})$$

$$\bar{\underline{L}} = \underline{R} \times \mu \underline{V}$$

Thus

$$\boxed{\underline{L} = \underline{r}_{cm} \times M \underline{v}_{cm} + \underline{R} \times \mu \underline{V}}$$

2) $T = \frac{1}{2} M v_{cm}^2 + \sum_i \frac{1}{2} m_i v_i^2 = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} m_1 \bar{\underline{v}}_1^2 + \frac{1}{2} m_2 \bar{\underline{v}}_2^2$

$$= \frac{1}{2} M v_{cm}^2 + \frac{1}{2} m_1 \bar{\underline{v}}_1^2 + \frac{1}{2} m_2 \left(\frac{m_1}{m_2}\right)^2 \bar{\underline{v}}_1^2$$

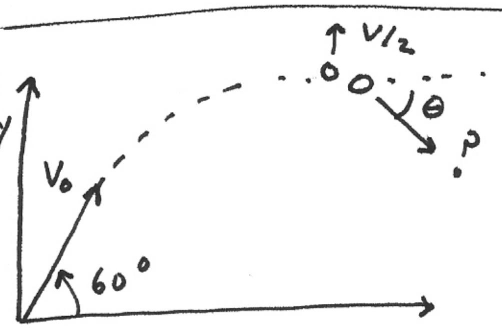
$$= \frac{1}{2} M v_{cm}^2 + \frac{1}{2} m_1 \left(1 + \frac{m_1}{m_2}\right) \bar{\underline{v}}_1^2$$

$$T = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} m_1 \left(1 + \frac{m_1}{m_2}\right)^{-1} \underline{V}^2$$

$$= \frac{1}{2} M v_{cm}^2 + \frac{1}{2} \left(\frac{m_1 m_2}{m_1 + m_2}\right) \underline{V}^2$$

$$\boxed{T = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} \mu \underline{V}^2}$$

3)



Immediately before explosion $v_y = 0$ and $v_x = v_0 \cos(60°) = \frac{1}{2} v_0$

$$\Rightarrow \quad \underline{P}_i = \frac{1}{2} m v_0 \hat{e}_x$$

$$\underline{P}_f = \underline{P}_i = \underbrace{\left(\frac{m}{2}\right)\left(\frac{v_0}{2}\right)\hat{e}_y}_{\text{frag } 1} + \underbrace{\frac{m}{2}\underline{V}}_{\text{frag } 2} = \underbrace{\frac{m}{2} v_0 \hat{e}_x}_{\text{Int. } \underline{P}}$$

$$\frac{m}{2} V = \frac{m}{2} v_0 \hat{e}_y - \frac{m}{4} v_0 \hat{e}_y$$

$$\underline{V} = v_0 \hat{e}_x - \frac{1}{2} v_0 \hat{e}_y$$

$$|V| = \left(1 + \frac{1}{4}\right)^{1/2} v_0 = 1.118\, v_0 \quad ; \quad \tan\theta = \frac{\frac{1}{2} v_0}{v_0} = \frac{1}{2} \quad \Rightarrow \theta = 26.6°$$

4) $v - v_0 = \Delta v = u \ln\left(\frac{m_0}{m}\right) \quad \Rightarrow \ln\left(\frac{m_0}{m}\right) = \frac{\Delta v}{u}$

$$\frac{m_0}{m} = e^{\Delta v / u} \qquad \text{or} \qquad m_0 = m\, e^{\Delta v / u}$$

$$m = m_0\, e^{-\Delta v / u} = m_0\, e^{-(3130/2600)} \simeq 0.3\, m_0$$

since m is the final mass after fuel is burned, $\Rightarrow$ ~ 70% of initial mass was fuel.

5) $v = \frac{dx}{dt} = -gt + u\ln\left(\frac{m_0}{m}\right)$

$dx = -gt\,dt + \left(u\ln m_0 - u\ln(m)\right)dt$ But $dt = -\frac{1}{\alpha}dm$

$\int_0^h dx = -g\int_0^t t\,dt - \frac{u}{\alpha}\ln(m_0)\int_{m_0}^{m} dm + \frac{u}{\alpha}\int_{m_0}^{m}\ln(m)\,dm$

$h = -\frac{1}{2}gt^2 - \frac{u}{\alpha}\ln(m_0)(m-m_0) + \frac{u}{\alpha}\left[m\ln m - m_0\ln m_0 - m + m_0\right]$

$h = -\frac{1}{2}gt^2 - \frac{u}{\alpha}(m_0-m) - \frac{um}{\alpha}\ln(m_0) + \cancel{\frac{um_0}{\alpha}\ln(m_0)}$

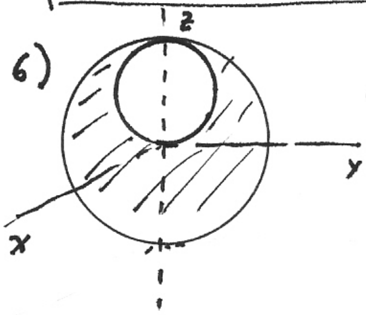
$+ \frac{um}{\alpha}\ln m - \cancel{\frac{um_0}{\alpha}\ln(m_0)}$

$h = -\frac{u}{\alpha}(m_0-m) - \frac{1}{2}gt^2 - \frac{um}{\alpha}\ln(m_0/m)$

since $\int_0^t dt = -\frac{1}{\alpha}\int_{m_0}^{m} dm \Rightarrow t = \frac{m_0-m}{\alpha}$

$$\boxed{h = -ut - \frac{1}{2}gt^2 - \frac{um}{\alpha}\ln(m_0/m)}$$

6)



obviously c.m. on z-axis, below origin.

consider prob to be a solid sphere of density ρ, radius a centered at origin plus a a sphere of mass density $-\rho$ & radius $a/2$ centered at $a/2$

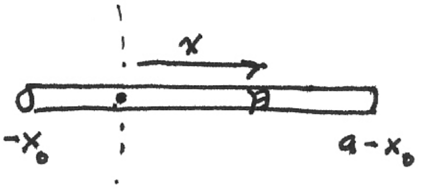
Mass of large sphere $M = \frac{4}{3}\pi a^3\rho$

Mass of small sphere $M_s = \frac{4}{3}\pi\left(\frac{a}{2}\right)^3(-\rho) = -\frac{1}{8}M$

Now $Z_{cm} = \frac{1}{M_{tot}}\sum_i z_i m_i$

$= \frac{1}{\left(M - \frac{1}{8}M\right)}\left[0M + \left(+\frac{a}{2}\right)\left(-\frac{1}{8}M\right)\right]$

$= -\left(\frac{8}{7M}\right)\left(\frac{a}{16}M\right) = \boxed{-\frac{a}{14}}$

7) 

$$I = \int_{-x_0}^{a-x_0} x^2 \rho \, dx$$

$$I = \frac{\rho}{3} x^3 \Big|_{-x_0}^{a-x_0} = \frac{\rho}{3}\left[(a-x_0)^3 - (-x_0)^3\right]$$

$$= \frac{\rho a^3}{3}\left[\left(1-\frac{x_0}{a}\right)^3 + \left(\frac{x_0}{a}\right)^3\right]$$

$$= \frac{1}{3} M a^2 \left[\left(1-\frac{x_0}{a}\right)^3 + \left(\frac{x_0}{a}\right)^3\right]$$

If $x_0 = 0$ $\quad I_0 = \frac{1}{3} M a^2$; $x_0 = \frac{a}{2}$; $I_c = \frac{1}{3} M a^2\left(\frac{1}{8}+\frac{1}{8}\right)$

$$I_c = \frac{1}{12} M a^2$$

8) $I_0 = \frac{2}{5} M a^2 \qquad L = I_0 \omega_0$

after collapse.

$$I_f = \frac{2}{5} M \left(\frac{a}{1000}\right)^2 = 10^{-6}\left(\frac{2}{5} M a^2\right) = 10^{-6} I_0$$

$$L = I_f \omega_f = I_0 \omega_0$$

$$\omega_f = \frac{I_0}{I_f}\omega_0 = \frac{I_0}{10^{-6} I_0}\omega_0 = 10^6 \omega_0$$

star Rotates 1 Million times faster.

If this was the sun.

$$\omega_0 = \left(\frac{2\pi \text{ rad}}{28 \text{ day}}\right)\left(\frac{1 \text{day}}{24 \text{hr}}\right)\left(\frac{1 \text{hr}}{3600 \text{ s}}\right) = 2.6 \times 10^{-6} \text{ s}$$

$\omega_f = 2.6$ sec.

$f = \frac{\omega}{2\pi} \qquad f_f = 0.41 \text{ s}^{-1}, \quad T_f = 2.41 \text{ s}$

Problem 6

$$-CV = m\frac{dV}{dt} + u\frac{dm}{dt}$$

$$\text{Let } \frac{dm}{dt} = -\alpha \quad \Rightarrow \quad dt = -\frac{dm}{\alpha}$$

$$-cV = m\frac{dv}{dt} - \alpha u \quad \Rightarrow \quad \alpha u - cV = m\frac{dv}{dt}$$

$$\int_{V_0}^{V} \frac{dV}{\alpha u - cV} = \frac{dt}{m} = -\frac{1}{\alpha}\int_{m_0}^{m} \frac{dm}{m}$$

↖ form 17.1.1

$$-\frac{1}{c}\ln(\alpha u - cV)\Big|_{V_0}^{V} = -\frac{1}{\alpha}\ln(m)\Big|_{m_0}^{m}$$

$$\ln(\alpha u - cV) - \ln(\alpha u - cV_0) = \frac{c}{\alpha}\left(\ln(m) - \ln(m_0)\right)$$

$$\ln\left(\frac{\alpha u - cV}{\alpha u - cV_0}\right) = -\frac{c}{\alpha}\ln\left(\frac{m_0}{m}\right) = +\ln\left(\frac{m_0}{m}\right)^{-c/\alpha}$$

$$\frac{\alpha u - cV}{(\alpha u - cV_0)} = \left(\frac{m_0}{m}\right)^{-c/\alpha}$$

$$\alpha u - cV = (\alpha u - cV_0)\left(\frac{m_0}{m}\right)^{-c/\alpha}$$

$$V = +\frac{1}{c}\left[\alpha u + (cV_0 - \alpha u)\left(\frac{m_0}{m}\right)^{-c/\alpha}\right]$$